

Depth-first Search of a Digraph (implemented using a stack)

Global variables and initializations:

- S*: A stack of vertices. The vertices on *S* will be the vertices on the path from the starting vertex up to and including the current vertex.
- adjList*: An array of lists. *adjList*[*v*] is the adjacency list of vertex *v*. For simplicity we allow the adjacency lists to be destroyed by the algorithm. *removeElement(adjList*[*v*]) will remove an element from *adjList*[*v*] and return it.
- color*: an array of vertices. *color*[*v*] will be the color of vertex *v*. Initially *color*[*v*] = *white* for all vertices.
- time*: relative time, initially 0.
- d*: an array of vertices. *d*[*v*] will be the “discover time” of vertex *v*, i.e., the time at which *v* is pushed onto the stack, colored gray, and preprocessed.
- f*: an array of vertices. *f*[*v*] will be the “finish time” of vertex *v*, i.e., the time at which *v* is postprocessed, colored black, and popped from the stack.

Depth first search of entire graph: Perform depth-first search from some white vertex, then from a second white vertex, a third, etc., until no white vertices remain.

```
void depthFirst( Digraph G)
    Initialize variables as above;
for ( each vertex  $v$  of  $G$  )
    if (  $color[v] == white$  )
        depthFirstFromVertex( $G, v$ );
return;
```

Depth first search from a single vertex: Performs depth-first search from a specific starting vertex.

```
void depthFirstFromVertex( Digraph  $G$ , Vertex  $start$ )
    discoverVertex(  $start$ );
while ( not empty( $S$ ) )
    if (  $adjList[top(S)]$  is nonempty )
         $adjacent = removeVertex( adjList[top(S)]$ );
        if (  $color[adjacent] == white$  )
            discoverVertex(  $adjacent$ );
        else if (  $color[adjacent] == gray$  )
            Process back edge ( $top(S), adjacent$ );
        else if (  $d[adjacent] < d[top(S)]$  )
            Process cross edge ( $top(S), adjacent$ );
        else
            Process forward edge ( $top(S), adjacent$ );
    else
        finishTopVertex();
return;
```

Discovering of a new vertex: Discover a new vertex x , adjacent to $top(S)$. The new vertex is pushed on the stack, colored gray, and its discover time is recorded. It undergoes any application-specific preorder processing.

```
void discoverVertex( Vertex x)
    push( S, x);
    color[top(S)] = gray;
    d[top(S)] = ++time;
    Preorder process vertex top(S);
return;
```

Exiting from the current vertex: Exit a vertex after all edges out of the vertex have been explored. Any application-specific postorder processing of the vertex is performed. The vertex is colored black, and its finish time is recorded. It is then popped from the stack.

```
void finishTopVertex( )
    Postorder process vertex top(S).
    f[top(S)] = ++time;
    color[top(S)] = black;
    pop(S)
return;
```

The running time of this algorithm is $\Theta(n+e)$, apart from the time for application-specific processing of the vertices and edges.