Recurrences — approximating ⌊*n*/2⌋ and ⌈*n*/2⌉ by n/2

If $C(n)$ denotes the number of comparisons used by mergesort in sorting a list with n elements (worst case), then

$$C(n) = n - 1 + C(\lfloor n/2 \rfloor) + C(\lceil n/2 \rceil), \quad C(1) = 0.$$

When n is a power of 2, we can write the recurrence as $C(n) = n - 1 + 2C(n/2)$, $C(1) = 0$, and the exact solution is $C(n) = n\lg(n) - n + 1$.

Is $n\lg(n) - n + 1$ a good approximation to $C(n)$ even when n is not a power of 2? The data below strongly suggests that the answer is yes.

n	$C(n)$	$n\lg(n)-n+1$	$\frac{C(n)}{n\lg(n)-n+1}$
3	3	2.8	1.0890
4	5	5.0	1.0000
5	8	7.6	1.0513
7	14	13.7	1.0255
10	25	24.2	1.0322
14	41	40.3	1.0173
20	69	67.4	1.0232
28	109	107.6	1.0130
40	177	173.9	1.0180
57	279	276.5	1.0091
81	440	433.5	1.0149
115	678	673.2	1.0071
164	1057	1043.6	1.0128
234	1617	1608.7	1.0052
334	2495	2467.2	1.0113
477	3782	3768.3	1.0036
682	5797	5739.1	1.0101
975	8727	8707.0	1.0023
1394	13287	13167.4	1.0091
1993	19876	19852.7	1.0012
2849	30093	29847.8	1.0082
4074	44793	44783.3	1.0002
5825	67534	67035.3	1.0074
8329	100223	100148.3	1.0007
11910	150357	149351.0	1.0067
17031	222698	222355.6	1.0015
24354	332543	330530.3	1.0061
34826	491681	490625.4	1.0022
49801	731281	727289.2	1.0055
71215	1079584	1076764.2	1.0026
101837	1600158	1592314.4	1.0049
145626	2359125	2352138.8	1.0030
208245	3486267	3471012.5	1.0044
297790	5133723	5117205.2	1.0032
425839	7566654	7537329.4	1.0039
608949	11130405	11092592.8	1.0034
870797	16367365	16311750.3	1.0034

For $2 \le n < 100$, maximum value of $C(n)\,/\,(n\lg(n)-n+1)$ is **1.0890** (at $n = 3$).

For $100 \le n < 1000$, maximum value of $C(n)\,/\,(n\lg(n)-n+1)$ is **1.0135** (at $n = 100$).

For $10^3 \le n < 10^4$, maximum value of $C(n)\,/\,(n\lg(n)-n+1)$ is **1.0091** (at $n = 1407$).

For $10^4 \le n < 10^5$, maximum value of $C(n)\,/\,(n\lg(n)-n+1)$ is **1.0069** (at $n = 11279$).

For $10^5 \le n < 10^6$, maximum value of $C(n)\,/\,(n\lg(n)-n+1)$ is **1.0052** (at $n = 180760$).