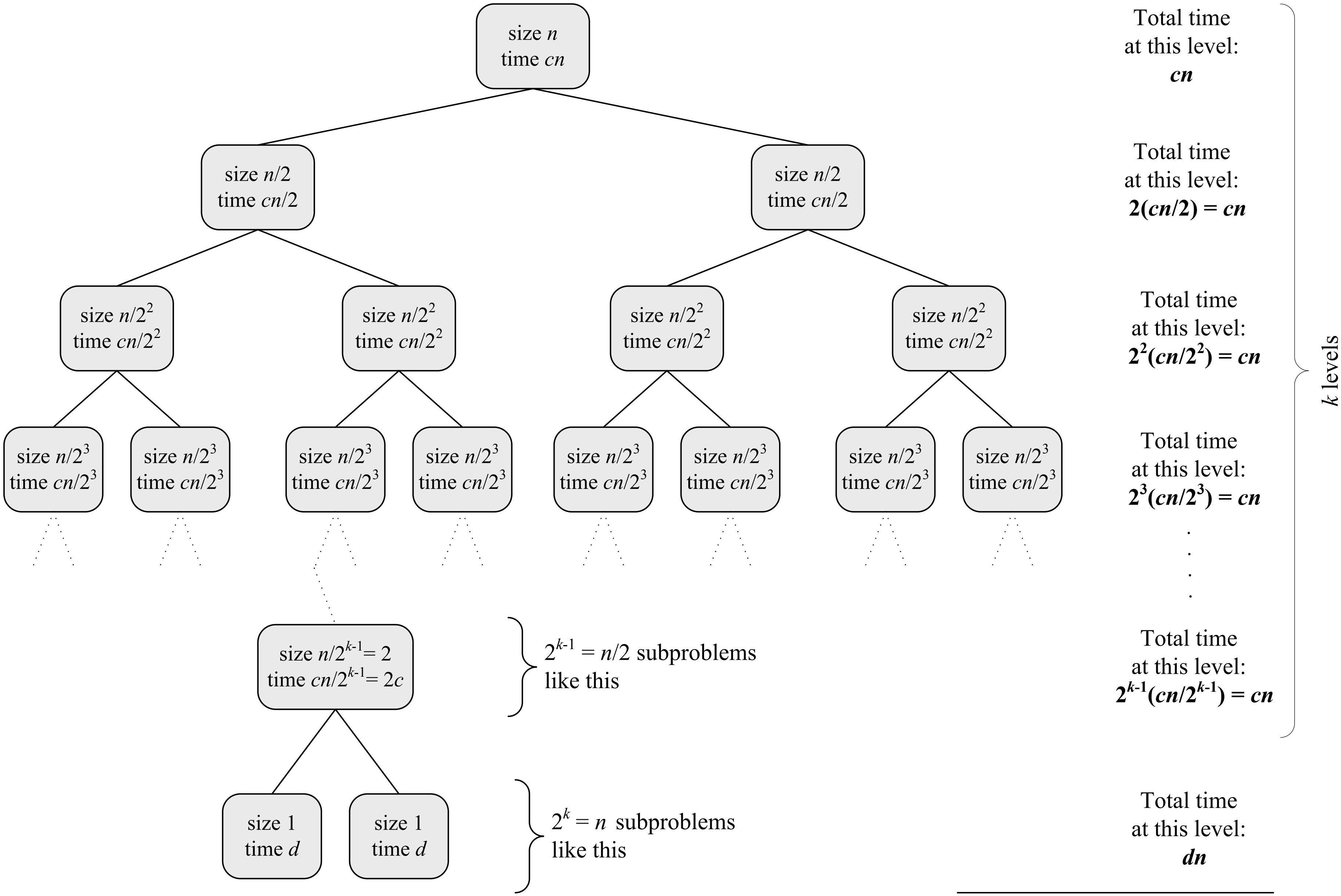
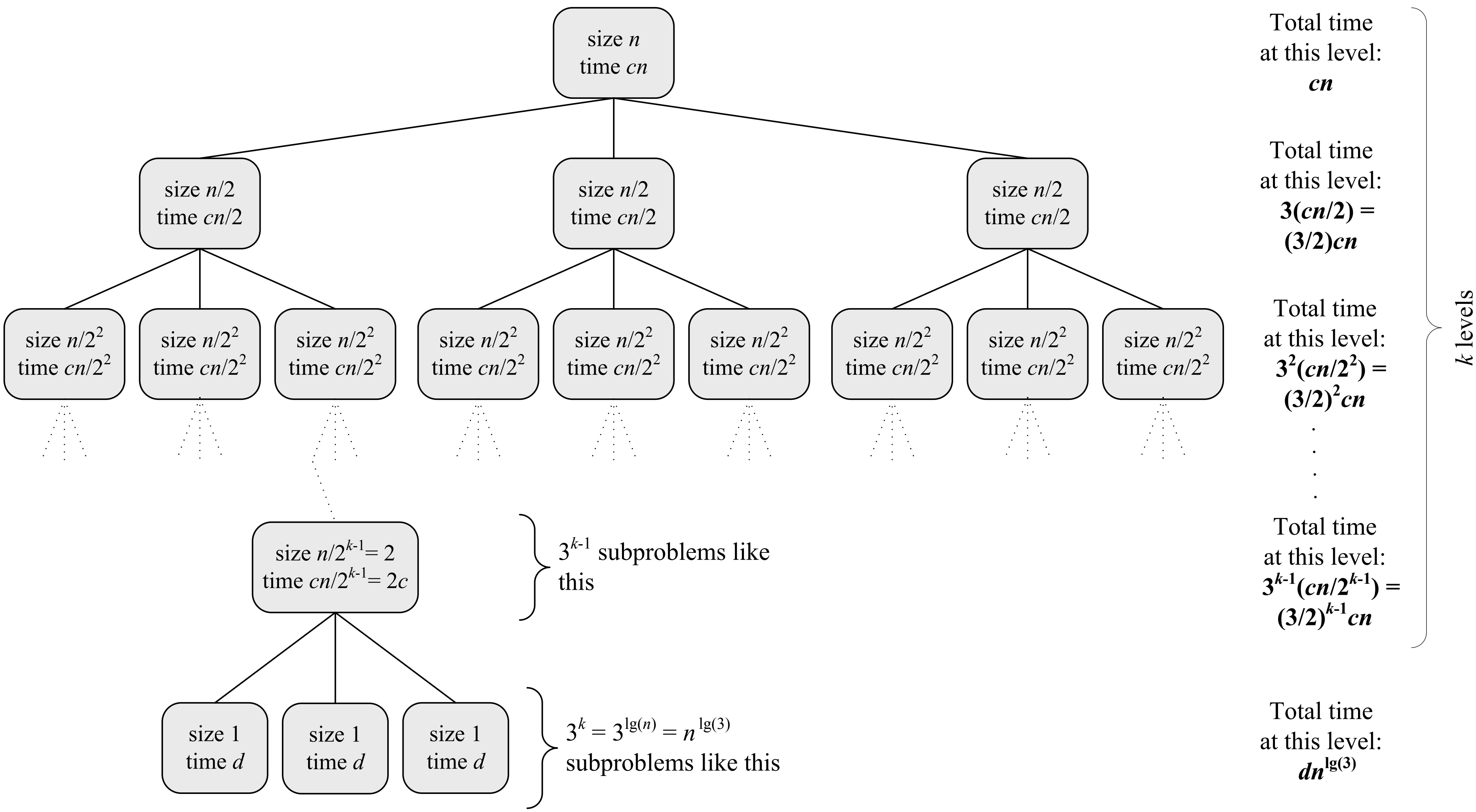
# The Recurrence $T(n) = 2T(n/2) + cn$, $T(1) = d$, when $n$ is a power of 2 ($n = 2^k$, $k = \lg(n)$)



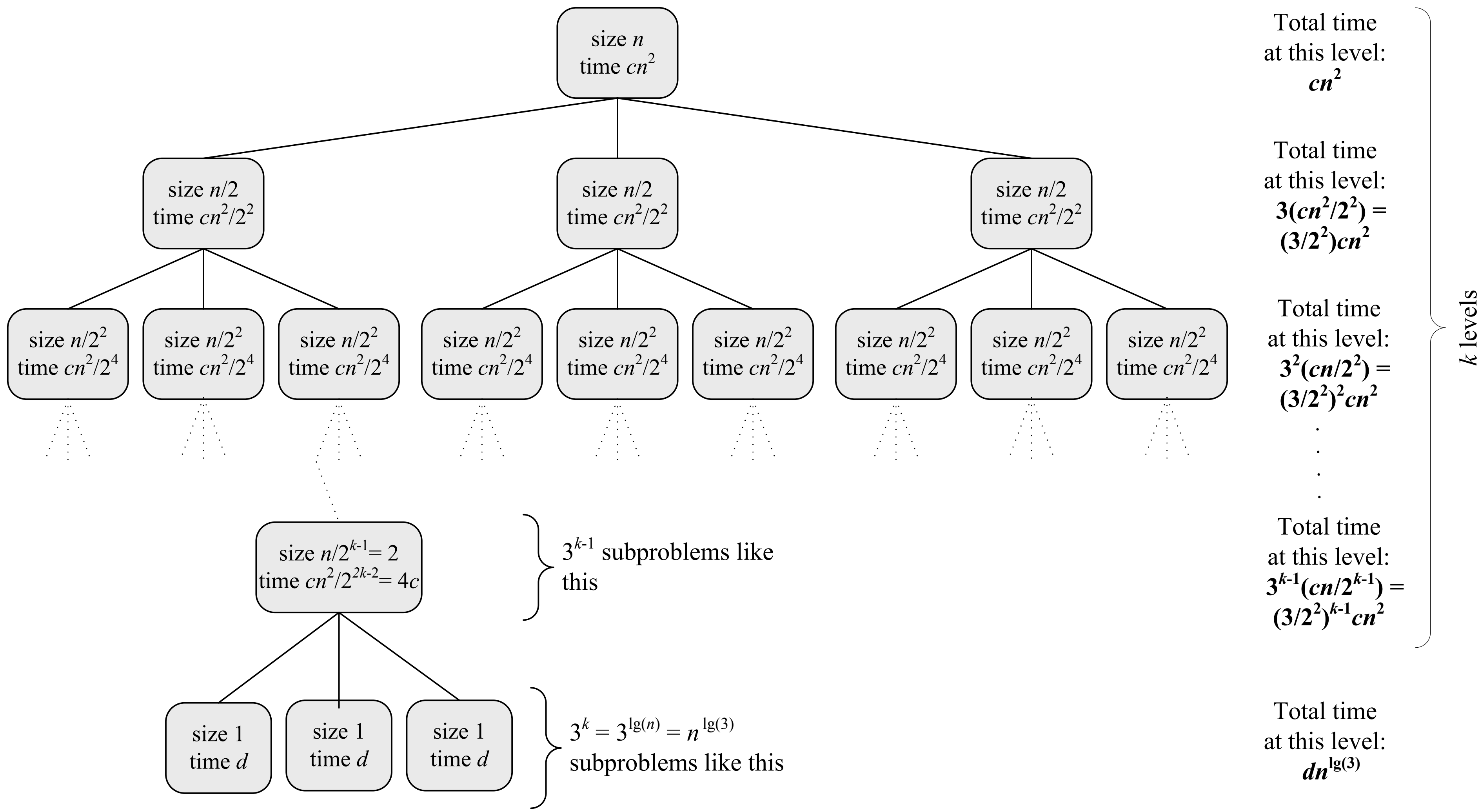

$$\begin{aligned} T(n) = \text{total time} &= k(cn) + dn \\ &= cn\lg(n) + dn \\ &= \Theta(n\lg(n)) \end{aligned}$$

# The Recurrence $T(n) = 3T(n/2) + cn$, $T(1) = d$, when $n$ is a power of 2 ($n = 2^k$, $k = \lg(n)$)



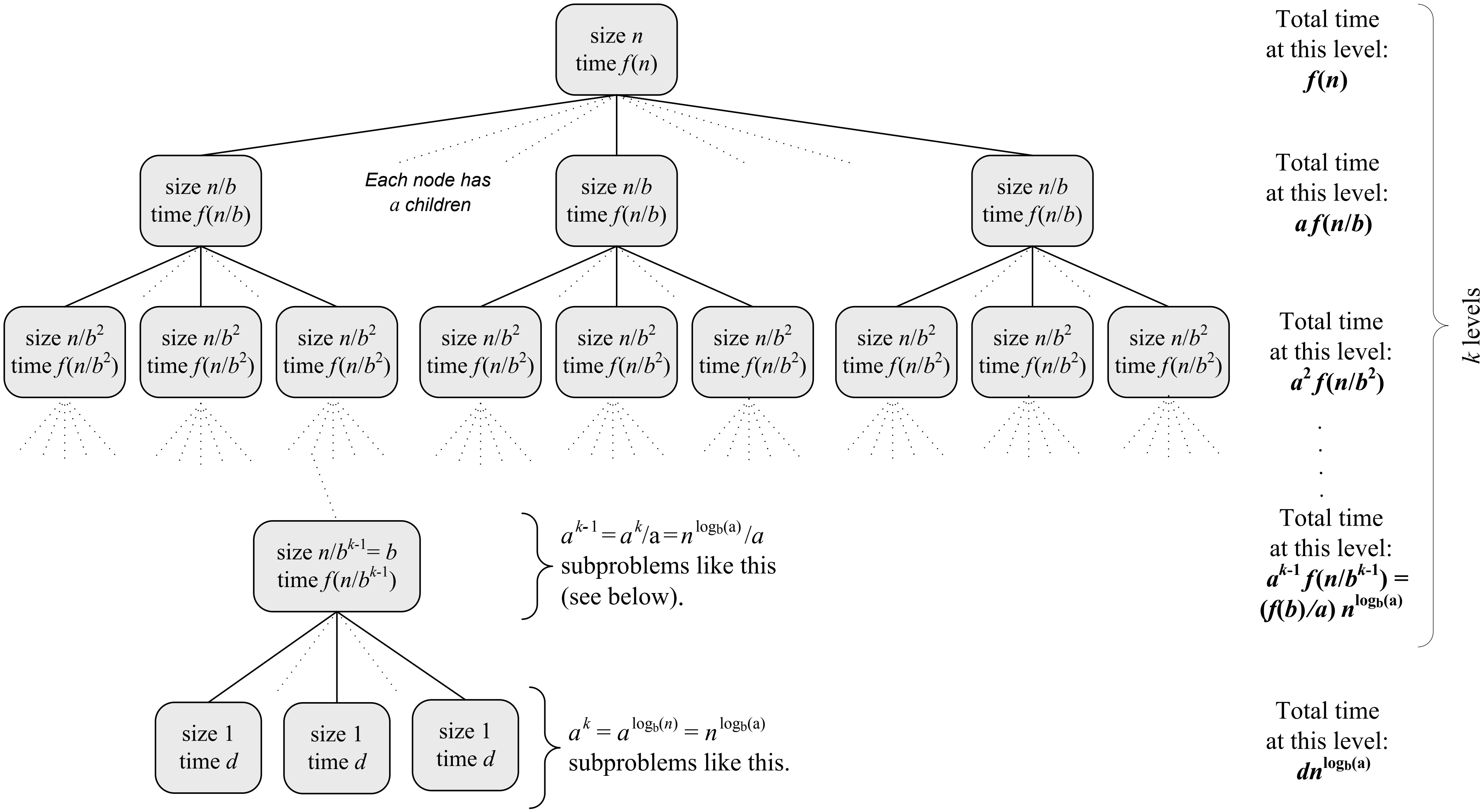

$$
\begin{aligned}
\mathrm{T}(n) = \text{total time} &= \left(1 + 3/2 + (3/2)^2 + \dots + (3/2)^{k-1}\right)cn + dn^{\lg(3)} \\
&= ((3/2)^k - 1)/(3/2 - 1)\ (cn) + dn^{\lg(3)} \\
&= (2c)(3^k/2^k)n - 2cn + dn^{\lg(3)} \\
&= (2c)3^k - 2cn + dn^{\lg(3)} \qquad (\text{since } n = 2^k) \\
&= \mathbf{(2c + d)\, n^{\lg(3)} - 2cn} \qquad (3^k = 3^{\lg(n)} = n^{\lg(3)}) \\
&= \mathbf{\Theta(n^{\lg(3)})}
\end{aligned}
$$

## The Recurrence $T(n) = 3T(n/2) + cn^2$, $T(1) = d$, when $n$ is a power of 2 ($n = 2^k$, $k = \lg(n)$)




$$
\begin{aligned}
T(n) = \text{total time} &= \left(1 + 3/2^2 + (3/2^2)^2 + \ldots + (3/2^2)^{k-1}\right)cn^2 + dn^{\lg(3)} \\
&= ((3/2^2)^k - 1)/(3/2^2 - 1)\ cn^2 + dn^{\lg(3)} \\
&= -4c(3^k/2^{2k})n^2 + 4cn^2 + dn^{\lg(3)} \\
&= -4c3^k + 4cn^2 + dn^{\lg(3)} && (\text{since } n^2 = 2^{2k}) \\
&= \mathbf{(d - 4c)\, n^{\lg(3)} + 4cn^2} && (3^k = 3^{\lg(n)} = n^{\lg(3)}) \\
&= \mathbf{\Theta(n^2)}
\end{aligned}
$$

# The Recurrence $T(n) = aT(n/b) + f(n)$, $T(1) = d$, when $n$ is a power of $b$ ($n = b^k$, $k = \log_b(n)$)




$$T(n) = \text{total time} = f(n) + a f(n/b) + a^2 f(n/b^2) + \ldots + a^{k-1} f(n/b^{k-1}) + dn^{\log_b(a)}$$