Name:__________________________________ **MATH 310 Quiz 1 (Sep 26, 2003)** no calculators!

(1) [3 points] Compute with any method you know: $\begin{vmatrix} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 3 & 4 & 4 & -3 \\ 0 & 2 & 3 & -1 \end{vmatrix} =$

(swap rows 1 and 2, then reduce with respect to row 1) $= - \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & 4 & 1 & -3 \\ 0 & 2 & 3 & -1 \end{matrix}$

(reduce w.r.t. row 2) $= \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 5 & 1 \end{matrix}$

(reduce w.r.t. row 3) $= \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 2 & -2 & -2 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$

(the determinant of a matrix containing a zero row) $= 0$

(2) [4 points] Let $\mathbf{x_1} = (1,2,3)^T$, $\mathbf{x_2} = (1,0,1)^T$, $\mathbf{y_1} = (-1,2,1)^T$, $\mathbf{y_2} = (0,0,1)^T$. Answer the following questions and explain your answers.

(a) Is $\mathbf{y_1} \in \mathrm{Span}(\mathbf{x_1}, \mathbf{x_2})$? **Yes.**

This question boils down to determining if the following system of linear equations is consistent:

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -2 & 4 \\ 0 & -2 & 4 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{array}\right)$$

The last augmented matrix represents a consistent system (therefore, its solution $(\alpha_1, \alpha_2)^T$ is such that $y_1 = \alpha x_1 + \alpha_2 x_2$).

(b) Is $\mathbf{y_2} \in \mathrm{Span}(\mathbf{x_1}, \mathbf{x_2})$? **No.**

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 3 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -2 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{array}\right)$$

The last row in the last matrix translates into the equation $0 = 1$, hence, the system is inconsistent.

(3) [3 points] Consider $S = \{A \in \mathbb{R}^{2\times 2} | \det(A) \neq 8.5\}$. Is S a subspace of the vector space $\mathbb{R}^{2\times 2}$ of 2×2 matrices? Explain.

There are many ways of showing that S is not a subspace. For instance, the following diagonal matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 8.5 \end{pmatrix} \in \mathbb{R}^{2\times 2}$$

has the determinant equal to 8.5, hence, does not belong to S. However,

$$A = B + C, \text{ where } B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & 0 \\ 0 & 8.5 \end{pmatrix}.$$

Both B and C belong to S, since $\det(B) = \det(C) = 0$; therefore, S is not closed under addition and is not a subspace.