- Section 1.1
  - Exercise 10.
  - Consider ideal $I = \langle xyz \rangle \subset R = k[x, y, z]$. Is I prime? If not, then is it possible to present it as an intersection of prime prime ideals? Is I radical?
- Section 1.2
  - Let M be an $m \times n$ matrix with real entries defining a monomial order $>_M$ in $k[x_1, ..., x_n]$ and having the following form:

$$M = \begin{pmatrix} & & & & 0 & 0 & ... & 0 \\ & N & & & 0 & 0 & ... & 0 \\ & & & & ... & ... & ... & ... \\ & & & & 0 & 0 & ... & 0 \\ * & * & ... & * & * & * & ... & * \\ * & * & ... & * & * & * & ... & * \\ ... & ... & ... & ... & ... & ... & ... & ... \\ * & * & ... & * & * & * & ... & * \end{pmatrix},$$

    where $N \in \mathbb{R}^{s \times t}$ $(s < m, t < n)$ is a matrix defining a monomial order $>_N$ in $k[x_1, ..., x_p]$. Show that $>_M$ is a monomial order eliminating variables $x_1, ..., x_p$.
- Section 1.3
  - Exercise 10
  - Exercise 12
  - Use the solution to the Exercise 12 to write an program in M2 that given an ideal $I \subset R = \mathbb{Q}[x, y, z]$ and a polynomial $h \in R$ computes a generating set for $I : \langle h \rangle$.
    Test the program on the input: $I = \langle x^2 + y^2 + z^2 - 1, zx, zy \rangle,\ h = z(z + 1)$.
  - Let the symmetric group S_n act on the set of variables $\{x_1, ..., x_n\}$ by permuting them. Then this action induces an action on the polynomial ring $R = k[x_1, ..., x_n]$: for $f \in R$ and $\sigma \in S_n$,

$$(\sigma \cdot f)(x_1, ..., x_n) = f(\sigma(x_1), ..., \sigma(x_n)).$$

    Consider the subset $R^{S_n} \subset R$ of polynomials invariant under the action of S_n. For example, the elementary symmetric functions,

$$\begin{aligned} s_1 &= x_1 + ... + x_n, \\ s_2 &= \sum_{1 \le i < j \le n} x_i x_j, \\ &... \\ s_n &= x_1 x_2 ... x_n, \end{aligned}$$

    belong to R^{S_n}.
    Show that for every $f \in R^{S_n}$ there exists a polynomial p such that $f = p(s_1, ..., s_n)$.
    (Hint: consider the degree lexicographic order on R and show that there exist $\alpha_1, ..., \alpha_n$ such that $LM(f) = LM(s_1^{\alpha_1} \cdot ... \cdot s_n^{\alpha_n})$.)
- Section 1.4
  - Exercise 11