

Varieties with components in several dimensions

Let $f = z^3 - 2zx^2 + y^2$.

Exercise. Determine the singular locus of $V(f)$; find Tjurina number at the singular point(s).

Consider the variety defined by the ideal $I = I_1 I_2$, where $I_1 = \langle f \rangle$ and $I_2 = \langle x, z \rangle$.

The two irreducible components in $V(I)$ are $V(I_1)$ and $V(I_2)$. The local analysis of the components can be done using standard bases technique.

Exercise. Find the local dimension of $V(I)$ at the origin and at the point $(1, 1, 1)$.

3D real plots in Maple

The following piece of Maple code produces the picture of $V(I)$ in the 3-dimensional real space.

```
> restart;
> with(plots): with(plottools):
> S := implicitplot3d(z^3-2*z*x^2+y^2, x=-1..1, y=-1..1, z=-1..1, grid=[40,40,40]):
> OR := sphere([0,0,0], 0.05, color=black):
> L := line([0,-1,0], [0,1,0], thickness=5, color=blue):
> display(S,OR,L, style=wireframe, orientation=[60,50]);
```

