

Review of the lectures 15 to 28

The exam is open book and open notes. To prepare for the exam you must organize your course materials to be ready for fast consultation. There will be no time to study the notes.

The questions below are just samples of the type of question you may expect.

1. Use `randomize` to reset the random number generator with the seed 111. Then use `rand()` to generate a list of 100 numbers. Call the list `l`.
 - (a) Replace every element x in the list `l` by $x \bmod 5$.
 - (b) Remove all duplicates from the list `l`.
 - (c) Sum the elements in the list.

2. Write an indexed procedure with name `rf`, which returns `rf[n](x)`, defined by

$$\mathbf{rf}[0](x) = 1, \mathbf{rf}[1](x) = x, \text{ and } \mathbf{rf}[n](x) = (x + 1)(\mathbf{rf}[n - 1](x) - \mathbf{rf}[n - 2](x)), \text{ for } n \geq 2.$$

The index n is the degree of the polynomial. Make sure the recursion runs efficiently.

3. What is a remember table in Maple? How is it used?
Give an example of a good use of a remember table.
4. Use the arrow operator to define the following operations on a polynomial p :
 - (a) remove all terms with negative coefficients from p ;
 - (b) replace x by x^3 in $p(x)$.

Use these two functions to define a function which does both operations to a polynomial.

5. Explain the difference between symbolic and automatic differentiation. Illustrate with an example the difference between the two and give the two Maple commands you need.

6. Consider the function $f(t) = \int_0^t (1 - e^t) dt$, for $t \geq 0$. Define this function in Maple.
What is $f'(0)$?

7. The function $g(x, t) = \frac{1-t^2}{1-2xt+t^2}$ is a generating function for the Chebyshev polynomials.
 - (a) Compute a Taylor series approximation for $g(x, t)$ around $t = 0$ of order 10. Select the coefficient of t^8 and compare with the output of `orthopoly[T](8, x)`. What is the difference between the two?
 - (b) Make a function `cp` in n (n is the degree of the Chebyshev polynomial) which uses this generating function and returns the same expanded polynomial as the one returned by `orthopoly[T](n, x)`. The function `cp` should work for any n , be careful for $n = 0$.

8. Consider $p(x) = 5x^2a^2 + 61x^2a + 66x^2 + 10xa^2 + 121xa + 121x + a^2 + 15a + 44$, as a polynomial in x with parameter a .
- Find the roots of p .
 - For which values of the parameter a is the answer valid?
 - Give the Maple command(s) to treat the special case(s).
 - As you can see the polynomial p is shown in expanded form. Give the Maple command to “un-expand”, i.e.: what is the command which reveals better the structure of p ?
9. How would you best solve for x the following expression:

$$-42 \sin(x)^{11} + 88 \sin(x)^8 - 76 \sin(x)^7 - 65 \sin(x)^5 + 25 \sin(x)^3 + 28$$

10. Consider the curve $x^4 - 3xy + y^4$. Give all Maple commands
- to make a plot for x and y both ranging between -2 and $+2$;
 - to convert the curve into polar coordinates; and
 - to plot the curve in polar coordinates.
11. Consider the system $\begin{cases} 1 + xy = 0 \\ 1 + x + y^2 = 0 \end{cases}$
- Draw the curves represented by the equations of the system. How many real solutions are there? Justify your answer.
 - Use `solve` to find all solutions, find approximations to the complex solutions of the system.
 - Solve the system using the Grobner basis approach.
12. Consider the initial value problem

$$x''(t) + 4x(t) = \sin(t), \quad x(0) = 1, x'(0) = 0.$$

- Give all Maple commands to define this problem and to solve it numerically.
 - Define a function which returns for every t the value of $x(t)$.
 - Plot the solution for t going from 0 to 10.
13. Give the Maple commands for the following tasks.
- Create a 5-by-5 matrix A where the (i, j) -the element is $\frac{1}{i+j}$ and a 5-by-1 matrix b of ones.

- (b) Find the eigenvalues and eigenvectors of A .
 - (c) Construct the augmented matrix $[A \ b]$ in order to solve the system $Ax = b$ with Gauss-Jordan elimination, i.e.: use `LinearAlgebra[ReducedRowEchelonForm]`.
14. Give an example of a good use of the `assume` command. What assumption should be made on x to be able to simplify $\sqrt{x^2 + 2x + 1}$ to $x + 1$?
15. Consider the integral $\int_0^t e^{\sin(x)} dx$. Give the Maple commands
- (a) to define this integral as a formula (call it `f`), without asking Maple to evaluate;
 - (b) to compute a 10-th order Taylor approximation (call it `T`) for it around $t = 0$;
 - (c) to use the Taylor approximation to create a function `F` which returns a number, e.g.: `F(2)` returns `12496/2835`.

Also review the homework assignments and make sure you understand the problems of the quizzes.