

Math 215. Homework 4

due 02/11/08

1. Prove that

a) $\{x \in \mathbf{R} \mid x^2 - 5x + 6 = 0\} = \{x \in \mathbf{Z} \mid 1 < x < 4\}$.

b) $\{x \in \mathbf{R} \mid x^2 - 5x + 6 < 0\} = \{x \in \mathbf{R} \mid x < 3\} \cap \{x \in \mathbf{R} \mid x > 2\}$

c) $\{x \in \mathbf{R} \mid x^2 - 5x + 6 > 0\} = \{x \in \mathbf{R} \mid x > 3\} \cup \{x \in \mathbf{R} \mid x < 2\}$

2. Prove that

a) $A \cap (A \cup B) = A$

b) $A \cup (A \cap B) = A$

Draw a Venn diagram to illustrate the proof. Present corresponding truth tables.

3. Let $A \Delta B = (A - B) \cup (B - A)$.

a) Show that $A \Delta B = (A \cup B) - (A \cap B)$

b) Show that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.

4. Let $X = \{x \in \mathbf{Z} \mid 5 \leq x \leq 10\}$. Find cardinality of $\mathcal{P}(X)$.

5. Give a proof or a counterexample for each of the following:

a) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}, x - y > 0$

b) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}, (x + y > 0 \text{ and } x + y = 0)$