Math 517. Spring 2009
Abstract Algebra. Midterm 2
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1. Find the degree of the extension $\mathbf{Q}(\sqrt{\mathbf{2}+\sqrt{\mathbf{2}}})/\mathbf{Q}$. Show that it is a Galois extension and determine its Galois group.

2. Determine the splitting field of the polynomial $x^p - x - a$ over $\mathbf{F}_p$ $(a \neq 0, a \in \mathbf{F}_p)$. Find the Galois group of the splitting field over $\mathbf{F}_p$.

3. Find a primitive element of $\mathbf{Q}(\sqrt{5}, \sqrt{17}, \sqrt{19})/\mathbf{Q}$

4. Find the degree of the extension $\mathbf{Q}(\zeta_{2^n})/\mathbf{Q}$ where ζ_{2^n} is a primitive root of unity $(n \geq 2)$. Find the degree and the Galois group of the extension $\mathbf{Q}(\zeta_{2^n} + \zeta_{2^n}^{-1})/\mathbf{Q}$.

5. Express $x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2$ as a polynomial in elementary symmetric functions.

6. a) Find the discriminant of the cylotomic polynomial $\Phi_{13}(x)$.
b) Show that $\mathbf{Q}(\sqrt{13}) \subset \mathbf{Q}(\zeta_{13})$.

7 Find the Galois group of the polynomial $t^4 + 4t + 2$.

8 Find the transcendence degree of $\mathbf{Q}(t, u, v, w)/\mathbf{Q}$ where $t^2 = 2$, u is transcendental over $\mathbf{Q}(t)$, $v^3 = t + 5$ and w is transcendental over $\mathbf{Q}(t, u, v)$.