1. Let $V \subseteq \mathbb{A}^n$ be an affine algebraic set and $f \in k[V]$. Show that the collection of points $(a_1, \ldots, a_n, f(a_1, \ldots, a_n)) \in \mathbb{A}^{n+1}$ such that $(a_1, \ldots, a_n) \in V$ is an affine algebraic set. Show that it is isomorphic to $V$.

2. Let $X = \text{Spec}R$ and $X_f$ be the principal open set corresponding to $f \in R$. Show that $X_f = X$ in and only if $f$ is a unit and that $X_f = \emptyset$ if and only if $f$ is nilpotent.

3. Let $R = \mathbb{Z}_{(2)}$ be the localization of $\mathbb{Z}$ at the prime ideal generated by 2. Let $M$ be $\mathbb{Q}$ considered as a $R$-module. Find the Jacobson radical of $R$, the number of generators of $M/2M$ and show that $M$ is not a finitely generated $R$-module.

4. Let $\mathbb{Z}_p$ be the ring of $p$-adic integers and let $M$ be the maximal ideal of $\mathbb{Z}_p$. For each $n \geq 0$ exhibit an element with valuation $n$ and find the dimension of $M^n/M^{n+1}$ as a vector space over the field $\mathbb{Z}_p/M$.

5. Let

$$
0 \to A \to B \to C \to 0
$$

$$
f \downarrow \quad g \downarrow \quad h \downarrow
$$

$$
0 \to A' \to B' \to C' \to 0
$$

be the commutative diagram. Prove that there is an exact sequence:

$$
\text{Ker} f \to \text{Ker} g \to \text{Ker} h \to \text{Coker} f \to \text{Coker} g \to \text{Coker} h
$$

(i.e. define the maps and show the exactness).

6. Let $R = k[x, y]$ where $k$ is a field and let $I = (x, y)$ be the ideal in $R$. Let $\alpha : R \to R^2$ be the map given by $\alpha(r) = (yr, -xr)$ and let $\beta : R^2 \to R$ be the map $\beta(r_1, r_2) = r_1x + r_2y$. Show that

$$
0 \to R \to R^2 \to R \to k \to 0
$$

where the map $R \to k = R/I$ is the canonical projection is a free resolution of $k$ as an $R$-module and show that $\text{Tor}_2^R(k, k) = k$.

7. Let $G = \mathbb{Z}/2\mathbb{Z}$ and $A = \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$. Compute $H^i(G, A)$ in the case when a) the action of $G$ on $A$ is trivial and b) non-trivial element of $G$ interchanges the factors of $A$. 

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