

MCS 521 – Combinatorial Optimization
Fall 2013
Problem Set 1

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Due: 9/24/13 at the beginning of class

Related reading: Appendix A

Instructions: Atop your problem set, please write your name and list your collaborators (see syllabus for the collaboration policy).

1. Giving details, derive Farkas' Lemma from Farkas' Lemma for Inequalities. (See Theorem A.1 and Corollary A.2 in book for definitions.) *Hint: the outline of the proof is in the textbook.*

2. Prove that $\exists x \geq 0$ s.t. $Ax \leq b$ iff $\forall y$ s.t. $y^T A \geq 0$ it is the case that $y^T b \geq 0$.

3. Assuming the sets are non-empty, prove that

a. $\max\{c^T x : Ax \geq b\} = \min\{y^T b : y \leq 0, y^T A = c^T\}$

b. $\max\{c^T x : x \geq 0, Ax \leq b\} = \min\{y^T b : y \geq 0, y^T A \geq c^T\}$