MCS 401 – Computer Algorithms I

Fall 2016

Problem Set 3

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Due: 10/12/16 by the beginning of class

Instructions: Atop your problem set, write your name and whether you are an undergraduate or graduate student. Also write the names of all the students with whom you have collaborated on this problem set.

1. [10 pts] Consider you get as input a very sparse undirected weighted graph $G = (V, E)$, in particular for which $|E| - |V| = 20$. Give an $O(|V|)$ time algorithm for finding a minimum spanning tree on G and prove your algorithm correct.

2. [10 pts] You are given two arrays, A and B, each of which contains n integers. The elements in each array are guaranteed to be in sorted order, i.e. $A[0] \leq A[1] \leq \ldots \leq A[n-1]$ and also $B[0] \leq B[1] \leq \ldots \leq B[n-1]$. Give as fast an algorithm as you can for finding the *median* value of all the $2n$ numbers in both A and B. (We define the median of $2n$ numbers to be the average of the nth smallest and nth largest values.) Argue that your algorithm is correct and give its running time.

3. [10 pts] You get as input n distinct positive integers $a_1, \ldots, a_n$. Give an $O(n \log n)$ algorithm to count the number of pairs $i < j$ where $a_i > 2a_j$.

4. [10 pts] You are given a $2^k \times 2^k$ board with one missing cell. Give an $O(2^{2k})$-time algorithm for filling the board with "L-shaped" tiles. (See Figure 1 below.)

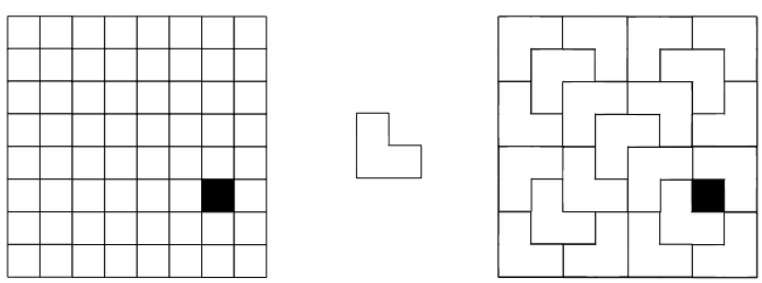

Figure 1: On the left is an example grid with a missing cell, with $k = 3$. In the middle is the "L-shaped" tile, to be used for tiling. On the right is an example solution.