1. [10 pts] Consider you get as input a very sparse undirected weighted graph \( G = (V, E) \), in particular for which \(|E| - |V| = 20\). Give an \( O(|V|) \) time algorithm for finding a minimum spanning tree on \( G \) and prove your algorithm correct.

2. [10 pts] You are given two arrays, \( A \) and \( B \), each of which contains \( n \) integers. The elements in each array are guaranteed to be in sorted order, i.e. \( A[0] \leq A[1] \leq \ldots \leq A[n - 1] \) and also \( B[0] \leq B[1] \leq \ldots \leq B[n - 1] \). Give as fast an algorithm as you can for finding the median value of all the \( 2n \) numbers in both \( A \) and \( B \). (We define the median of \( 2n \) numbers to be the average of the \( n \)th smallest and \( n \)th largest values.) Argue that your algorithm is correct and give its running time.

3. [10 pts] You get as input \( n \) distinct positive integers \( a_1, \ldots, a_n \). Give an \( O(n \log n) \) algorithm to count the number of pairs \( i < j \) where \( a_i > 2a_j \).

4. [10 pts] You are given a \( 2^k \times 2^k \) board with one missing cell. Give an \( O(2^{2k}) \)-time algorithm for filling the board with “L-shaped” tiles. (See Figure 1 below.)

Figure 1: On the left is an example grid with a missing cell, with \( k = 3 \). In the middle is the “L-shaped” tile, to be used for tiling. On the right is an example solution.