MCS 401 – Computer Algorithms I

Fall 2019

Problem Set 5

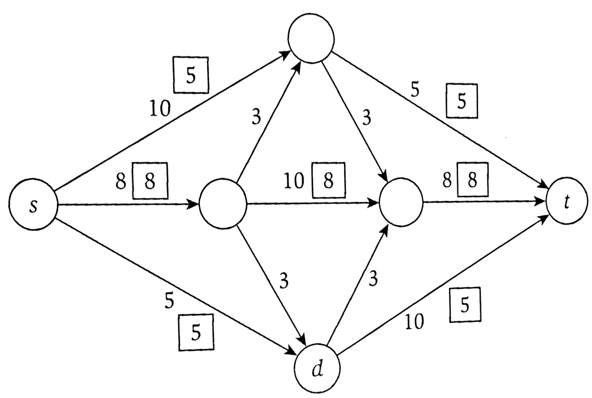
Lev Reyzin

Due: 11/11/19 by the beginning of class

Instructions: Atop your problem set, write your name and whether you are an undergraduate or graduate student.

1. Let G be an arbitrary flow network with a source s, sink t, and positive integer capacity c_e on every edge e. Let (A, B) be a minimum s–t cut of G with respect to these capacities $\{c_e : e \in E\}$. Now, if we add 1 to every capacity, will (A, B) remain a minimum s–t cut with respect to the new capacities $\{1 + c_e : e \in E\}$? Prove your answer correct.

2. Consider the network below in which an s–t flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in the boxes give the amount of flow sent on each edge. (Edges without numbers in boxes have no flow being sent on them.)



What is the value of this flow? Is this a maximum (s, t) flow in the graph? What is the minimum s–t cut in this graph? (It is suggested that you use this problem to practice your understanding of the Ford-Fulkerson algorithm)

3. Show that for every directed graph $G = (V, E)$ with edge capacities that are positive integer multiples of 3 and for every pair of vertices s and t (where $s \neq t$ and t is reachable from s), the value of the maximum $s-t$ flow must be an integer multiple of 3.

4. You are given an undirected graph $G = (V, E)$. A certain collection of nodes $D \subset V$ are designated as *dangerous* and a certain collection of nodes $S \subset V$ is designated as *safe* (assume $D \cap S = \emptyset$). In an emergency, we want a set of evacuation routes from the dangerous nodes to the safe ones. A set of evacuation routes is defined as a set of paths in G such that:

i. each node in D is the tail of one path,

ii. the head of each path lies in S,

iii. no two paths share any nodes in common.

Such a collection lets the populations of D escape to S without overly congesting any edge in G.

Given G, D, and S, give a polynomial time algorithm to decide if a set of evacuation routes exists in G.

5. Is the following claim true or false? *"For every directed graph $G = (V, E)$ with positive integer capacities and for every pair of vertices s and t (where $s \neq t$ and t is reachable from s), there always exists an edge such that increasing the capacity of that edge will increase the maximum s–t flow that is possible in G."* Explain your answer.

6. Let $G = (V, E)$ be a directed graph with a source $s \in V$, sink $t \in V$, and positive integer edge capacities $\{c_e\}$. Give a polynomial time algorithm to decide whether G has *more than one* distinct minimum s–t cut.