

MCS 549 – Foundations of Data Science
Fall 2021
Problem Set 2

Lev Reyzin

Due: 11/1/21 at the beginning of class

Instructions: Atop your problem set, please write your name and list your collaborators.

Problems

1. Given the set of integers $\{1, 2, \dots, n\}$, what is the expected number of draws d with replacement until the integer 1 is drawn? What is the expected number of draws until every integer from the set is drawn? (This is needed for the expected cover time of K_n .)
2. What is the hitting time h_{uv} for two adjacent vertices on a cycle of length n ? What is the hitting time if edge (u, v) is removed?
3. What is the escape probability of a random walk starting at the root of an infinite binary tree.
4. Find the threshold for $p(n)$ for the existence of 4-cliques in $G(n, p(n))$. Prove your answer correct.
5. In class we showed that if the degrees in $G(n, \frac{1}{n})$ were independent, there would be a vertex of degree

$$d = \frac{\log n}{\log \log n}$$

with constant positive probability. However, the degrees are not independent. Show how to overcome this difficulty.

6. Show that in $G(n, 1/2)$ there are almost surely no cliques of size greater than or equal to $2 \log_2 n$. Then, use the second moment method to show that in $G(n, 1/2)$, almost surely there are cliques of size $(2 - \varepsilon) \log_2 n$ (for any constant $\varepsilon > 0$).