STAT 473 – Game Theory
Fall 2021
Problem Set 4

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Due: 11/23/21, 9:30 am

1. [10 pts] Suppose that you and a friend are dividing 4 gold coins, and each of you want to maximize the number of gold coins that you get. You two decide on the following procedure: first you will divide the coins into two piles, and then your friend will choose the pile he wants, leaving the other pile for you. Draw the game tree for this extensive form game. What is the equilibrium strategy for both players? How will the coins end up being divided if both players play optimally[1]?

2. [10 pts] Consider the Centipede Game (Example 6.1.6). In class we saw that the pair of strategies where both players always defect are in Nash equilibrium. In fact, both players always defecting is a special type of equilibrium called a "subgame perfect Nash equilibrium"[2] because at each stage of the game, defecting is the equilibrium strategy for that subgame. However, there are many other pure strategy Nash equilibria. What are all these other pure equilibria? (And why?)

3. [10 pts] In the Fish Seller Game (Example 6.3.1), suppose that the seller isn't perfect and can only tell whether his fish is fresh or old with probability 0.9. Draw the game tree for this Bayesian game and determine the normal-form representation of the game.

4. [10 pts] Consider the Iterated Prisoner's Dilemma with payoffs as in section 6.4 in the "discounted payoff" setting. The Grim strategy is the following: Cooperate until a round in which the other player defects, and then defect from that point on. Determine for which values of β it is a Nash equilibrium in Iterated Prisoner's Dilemma for both players to use the Grim strategy.

5. [10 pts] Consider the following simplified game between the USSR (player 1) and the USA (player 2). First, the USSR needs to decide whether to launch a nuclear attack on the USA or not. If the USSR decides not to attack, then the payoffs are $(0, 0)$. If the USSR does attack, the USA needs to decide whether to retaliate with a nuclear strike of its own. If the USA does not retaliate, then the payoffs are $(100, -10000)$. If the USA does retaliate, the payoffs are $(-M, -M)$, where M is a huge number ($M >> 10000$) whose negation corresponds to the end of life on Earth. Write the game tree of this extensive-form game. Use backwards induction to find the resulting outcome if both players play according to their best strategies. Open-ended: what could the USA do to potentially change the game to avoid this bad outcome?

[1] This is a zero-sum game, so it makes sense to talk about optimal values.
[2] This is the equilibrium found by backward induction.