MCS 441 – Theory of Computation I
Spring 2013
Problem Set 2

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Due: 2/1/13 at the beginning of class

Related reading: Chapter 1.1
Instructions: Atop your problem set, write your name, clearly list your collaborators[1] (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.
Important note: Problems labeled "U" and "G" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

Drawing state machines

1. **[9 pts]** Draw state diagrams for DFAs recognizing the following languages:

i. $L_1 = \{w \mid \text{length of } w \text{ is odd}\}$, $\Sigma = \{1\}$

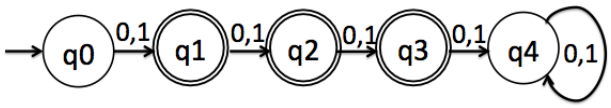
ii. $L_2 = \{w \mid w \text{ begins with "}aaa\text{" or ends with "}aaa\text{"}\}$, $\Sigma = \{a, b\}$. Restriction: your DFA may contain no more than 8 states.

iii. $L_3 = \{w \mid \text{the characters of } w, \text{ interpreted as decimal digits, sum to less than } 6\}$, $\Sigma = \{1, 2, 3\}$.

Reading state machines

2. **[10 pts]** For each of the following DFAs, explain what language they recognize:

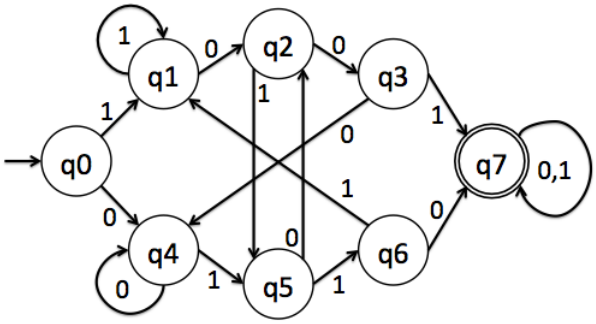
i. M_1



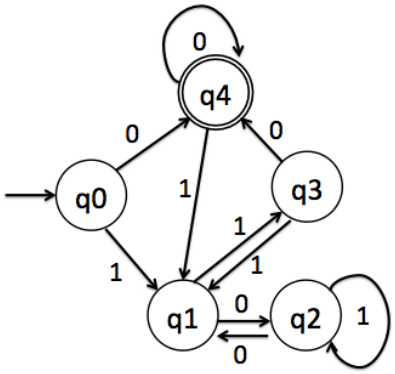
For machine M_1, also give its formal description as a 5-tuple. You do not need to do this for the machines that follow in parts ii. and iii. of this question.

[1] If you did not have any collaborators, please say so.

ii. M_2



iii. M_3



Closure

3. **[6 pts]** Let A and B be regular languages. Show that $A \backslash B$ is also regular. (Remember that $A \backslash B = \{x | x \in A, x \notin B\}$. Hence, this operation removes all strings from A that are also in B.)

Counting

4. **[9 pts]** In this problem we shall examine some limitations behind small automata. Note that a DFA is allowed to have unreachable states.

i. How many different languages can be recognized by 2 state DFAs over $\Sigma = \{1, 2\}$?

ii. Give an example of a regular language that cannot be recognized by a 3 state DFA. Explain why.

iii. **U.** Describe all the languages recognizable by 1 state DFAs over $\Sigma = \{1\}$.

G. Give an upper bound on the number of different languages recognizable by an n state machine over an alphabet of size s, as a function of n and s. Explain why your bound is valid.