Related reading: Chapter 1.1
Instructions: Atop your problem set, write your name, clearly list your collaborators\(^{\dagger}\)(see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.
Important note: Problems labeled “U” and “G” are assigned to undergraduate or graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

Drawing state machines
1. [9 pts] Draw state diagrams for DFAs recognizing the following languages:
   i. \(L_1 = \{w \mid \text{length of } w \text{ is odd}\}, \Sigma = \{1\}\)
   ii. \(L_2 = \{w \mid w \text{ begins with “aaa” or ends with “aaa”}\}, \Sigma = \{a, b\}\). Restriction: your DFA may contain no more than 8 states.
   iii. \(L_3 = \{w \mid \text{the characters of } w, \text{interpreted as decimal digits, sum to less than } 6\}, \Sigma = \{1, 2, 3\}\).

Reading state machines
2. [10 pts] For each of the following DFAs, explain what language they recognize:
   i. \(M_1\)

   \[\begin{array}{c}
   q_0 \xrightarrow{0,1} q_1 \xrightarrow{0,1} q_2 \xrightarrow{0,1} q_3 \xrightarrow{0,1} q_4 \xrightarrow{0,1}
   \end{array}\]

   For machine \(M_1\), also give its formal description as a 5-tuple. You do not need to do this for the machines that follow in parts ii. and iii. of this question.

\(^{\dagger}\)If you did not have any collaborators, please say so.
ii. $M_2$

iii. $M_3$

Closure

3. [6 pts] Let $A$ and $B$ be regular languages. Show that $A \setminus B$ is also regular. (Remember that $A \setminus B = \{ x | x \in A, x \notin B \}$. Hence, this operation removes all strings from $A$ that are also in $B$.)

Counting

4. [9 pts] In this problem we shall examine some limitations behind small automata. Note that a DFA is allowed to have unreachable states.

   i. How many different languages can be recognized by 2 state DFAs over $\Sigma = \{1, 2\}$?

   ii. Give an example of a regular language that cannot be recognized by a 3 state DFA. Explain why.

   iii. Describe all the languages recognizable by 1 state DFAs over $\Sigma = \{1\}$.

   G. Give an upper bound on the number of different languages recognizable by an $n$ state machine over an alphabet of size $s$, as a function of $n$ and $s$. Explain why your bound is valid.