MCS 441 – Theory of Computation I

Spring 2013

Problem Set 3

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Due: 2/8/13 at the beginning of class

Related reading: Chapters 1.1-1.3, focusing on 1.2.
Instructions: Atop your problem set, write your name, clearly list your collaborators[1] (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.

NFA Design

1. [6 pts] Give state diagrams for any NFAs recognizing the following languages over $\Sigma = \{0, 1\}$.

i. [3 pts] $L_1 = \{w \mid w \text{ contains two consecutive 1s or } w \text{ contains no 0s}\}$

ii. [3 pts] $L_2 = \{w \mid w = w_1 w_2 \ldots w_n \text{ with } w_{n-3} = 1 \text{ and } w_{n-1} = 0\}$

NFAs and DFAs

2. [10 pts] Consider the NFA: $N = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$, with δ defined in Table 1.

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\{q_1\}$	$\{q_1\}$

Table 1: The transition function δ for N

i. [3 pts] Draw the state diagram for N.

ii. [3 pts] What language does N recognize?

iii. [3 pts] Let M_1 be a DFA recognizing $L(N)$. Using the "power set" construction in the proof of Theorem 1.39 from Sipser, draw the state diagram for M_1, labeling the states of M_1 with the corresponding members of $\mathcal{P}(\{q_1, q_2\})$.

iv. [1 pts] Let M_2 be a DFA recognizing $L(M_1)$ but containing fewer states than M_1. Draw the state diagram of M_2.

[1] If you did not have any collaborators, please say so.

Accept States

3. [6 pts] Remember that GNFAs may have only one accept state but can still recognize any regular language.

i. [3 pts] If we allowed NFAs to have only one accept state, would they still be able to recognize any regular language? Why or why not?

ii. [3 pts] How about DFAs? Why or why not?

More Closure

4. [6 pts] For a string $w = w_1w_2 \ldots w_n$, let $w^{\leftrightarrow} = w_nw_{n-1} \ldots w_1$; further, let $\epsilon^{\leftrightarrow} = \epsilon$. For a language A, define the operation

$$A^{\leftrightarrow} = \{w^{\leftrightarrow} \mid w \in A\}.$$

Show that A is regular if and only if $A^{\leftrightarrow}$ is regular.

Representation

5. [12 pts] This question explores the conciseness of representation of regular languages.

i. [1 pt] Argue that if a language can be recognized by a DFA with k states then it can also be recognized by an NFA with k states.

Let $\Sigma^n = \underbrace{\Sigma\Sigma \ldots \Sigma}_{n}$. Consider the regular language $R_1 = \Sigma^*1\Sigma^{k-1}$ over $\Sigma = \{0, 1\}$.

ii. [2 pts] Show that R_1 can be recognized by an NFA with $k + 1$ states.

iii. [6 pts] Prove that any DFA that recognizes R_1 must have at least 2^k states.

You can get full credit for the next questions even if you were not able to answer parts i. – iii.

iv. [1 pt] What does part iii. of this question tell you about Theorem 1.39 from Sipser?

v. [2 pts] What do parts i., ii., and iii. of this question tell you about DFAs as compared to NFAs? Be concrete.