MCS 441 – Theory of Computation I
Spring 2013
Problem Set 4

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Due: 2/18/13 at the beginning of class

Related reading: Chapters 1.1-1.4, focusing on 1.3 and 1.4.
Instructions: Atop your problem set, write your name, list your collaborators[1] (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.
Note: Answer the questions in order that they are numbered on the problem set.

1. [6 pts] Let $\Sigma = \{0, 1\}$. Give regular expressions for the following languages.

a. [2 pts] $L_{1a} = \{w \mid$ every even position of w has a $0\}$

b. [2 pts] $L_{1b} = \{w \mid w$ contains at least four 1s$\}$

c. [2 pts] $L_{1c} = \{w \mid w$, interpreted as a binary number, is divisible by 2 (or 10 in binary)$\}$

Note that $\varepsilon \in L_{1a}$ and $\varepsilon \notin L_{1b}, L_{1c}$.

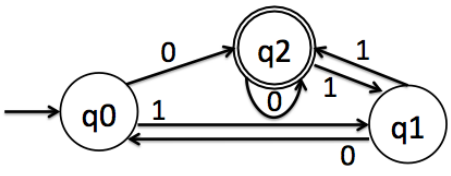
2. [9 pts] Let $\Sigma = \{0, 1\}$. Convert the following regular expressions to NFAs recognizing the same language. Draw the state diagrams for the NFAs.[2]

a. [3pts] $R_{2a} = \Sigma^* 11 \Sigma^*$

b. [3pts] $R_{2b} = ((11)^* 00 \cup 01)^*$

c. [3pts] $R_{2c} = \emptyset^*$

3. [4 pts] Convert the following DFA, M_3, to a regular expression recognizing the same language.[3]



[1] If you did not have any collaborators, please say so.
[2] You may use Lemma 1.55 or solve these some other way.
[3] You may use Lemma 1.60 or solve this question some other way.

4. [9 pts] Use the Pumping Lemma to show the following languages are not regular.

a. [3 pts] $L_{4a} = \{www \mid w \in \Sigma^*\}$, $\Sigma = \{0, 1\}$.

b. [3 pts] $L_{4b} = \{w \mid w = w^{\leftrightarrow} \text{ and } w \in \Sigma^*\}$, $\Sigma = \{0, 1\}$. Remember, in Problem Set 3 for $n \geq 1$ and $w = w_1 w_2 \dots w_n$, we defined $w^{\leftrightarrow} = w_n w_{n-1} \dots w_1$ and $\epsilon^{\leftrightarrow} = \epsilon$.

c. [3 pts] $L_{4c} = \{1^{2^n} \mid n \geq 1\}$, $\Sigma = \{1\}$.

5. [5 pts] Let $\Sigma = \{q, r, s\}$. Consider the language:

$$L_5 = \{q^i r^j s^k \mid i, j, k \geq 0 \text{ and } (i = 1) \rightarrow (j = k)\}.$$

a. [3 pts] Does L_5 satisfy the conditions of the pumping lemma? Why or why not?

b. [2 pts] What does the answer to a. imply about L_5?

6. [4 pts] Let $\Sigma = \{0, 1\}$. Consider the language:

$$L_6 = \{1^n x 1^n \mid n \geq 1, x \in \Sigma^*\}.$$

Is the language L_6 regular or not? Prove your answer correct.

7. [5 pts] Prove or give a counterexample to the following claim: language L_7^* is regular if and only if language L_7 is regular.