1. [6 pts] Let \( \Sigma = \{0, 1\} \). Give regular expressions for the following languages.
   a. [2 pts] \( L_{1a} = \{ w \mid \text{every even position of } w \text{ has a } 0 \} \)
   b. [2 pts] \( L_{1b} = \{ w \mid w \text{ contains at least four } 1s \} \)
   c. [2 pts] \( L_{1c} = \{ w \mid w, \text{ interpreted as a binary number, is divisible by } 2 \text{ (or } 10 \text{ in binary)} \} \)
   Note that \( \varepsilon \in L_{1a} \) and \( \varepsilon \notin L_{1b}, L_{1c} \).

2. [9 pts] Let \( \Sigma = \{0, 1\} \). Convert the following regular expressions to NFAs recognizing the same language. Draw the state diagrams for the NFAs.
   a. [3 pts] \( R_{2a} = \Sigma^*11\Sigma^* \)
   b. [3 pts] \( R_{2b} = ((11)^*00 \cup 01)^* \)
   c. [3 pts] \( R_{2c} = \emptyset^* \)

3. [4 pts] Convert the following DFA, \( M_3 \), to a regular expression recognizing the same language.

---

1If you did not have any collaborators, please say so.
2You may use Lemma 1.55 or solve these some other way.
3You may use Lemma 1.60 or solve this question some other way.
4. [9 pts] Use the Pumping Lemma to show the following languages are not regular.
   a. [3 pts] \( L_{4a} = \{ w w w \mid w \in \Sigma^* \}, \Sigma = \{0, 1\} \).
   b. [3 pts] \( L_{4b} = \{ w \mid w = w^{\leftrightarrow} \text{ and } w \in \Sigma^* \}, \Sigma = \{0, 1\} \). Remember, in Problem Set 3 for \( n \geq 1 \) and \( w = w_1 w_2 \ldots w_n \), we defined \( w^{\leftrightarrow} = w_n w_{n-1} \ldots w_1 \) and \( \epsilon^{\leftrightarrow} = \epsilon \).
   c. [3 pts] \( L_{4c} = \{ 1^{2^n} \mid n \geq 1 \}, \Sigma = \{1\} \).

5. [5 pts] Let \( \Sigma = \{q, r, s\} \). Consider the language:
   \( L_5 = \{ q^i r^j s^k \mid i, j, k \geq 0 \text{ and } (i = 1) \rightarrow (j = k) \} \).
   a. [3 pts] Does \( L_5 \) satisfy the conditions of the pumping lemma? Why or why not?
   b. [2 pts] What does the answer to a. imply about \( L_5 \)?

6. [4 pts] Let \( \Sigma = \{0, 1\} \). Consider the language:
   \( L_6 = \{ 1^n x 1^n \mid n \geq 1, x \in \Sigma^* \} \).
   Is the language \( L_6 \) regular or not? Prove your answer correct.

7. [5 pts] Prove or give a counterexample to the following claim: language \( L_7^* \) is regular if and only if language \( L_7 \) is regular.