# MCS 441 – Theory of Computation I
## Spring 2013
## Problem Set 6

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**Due**: 3/8/2013 at the beginning of class

**Related reading:** Chapter 3
**Instructions:** Atop your answers, write your name, clearly list your collaborators[1] (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.

**1. [7 pts]** Let $L_1$ be the language $\{0^n 1^n \mid n \geq 1\}$ over $\Sigma = \{0, 1\}$.

a. [5 pts.] Draw the state diagram for $M_1$, a Turing Machine that recognizes $L_1$. It will be assumed that all the transitions not depicted go to the reject state, which may or may not be drawn.
Note: knowing how to explicitly program TMs is not an important skill. However, doing it once is a worthwhile exercise.

b. [2 pts] Is $M_1$, as constructed in the previous part, a decider?
Note: you might have constructed a machine for which it is very hard to answer this question. In that case, consider changing your machine.

**2. [6 pts]** Read the Introduction, Sections 1 and 2, and Section 9 up to the end of 9.I of Alan Turing's 1936 paper: `http://homepages.math.uic.edu/~lreyzin/s13_mcs441/Turing36.pdf`.

a. [3 pts] Give an example of a computation that Turing would have probably believed a human would be unable to do without paper. Justify your answer.

b. [3 pts] Is Turing more concerned with creating a machine that can simulate human computation or with creating a machine that humans can simulate? Justify your answer.

Note: answer the two questions above briefly, referring to and/or quoting the text for support. *Do not write long essays answering these questions – a well thought-out paragraph should be sufficient for each.*

**3. [5 pts]** Imagine a modified Turing Machine that has a tape that is infinitely long in *both* directions. The machine starts at the beginning of the input, and the tape has blank space going infinitely both before the beginning and after the end of the input. Are there any languages that this machine can recognize that a Turing Machine cannot? Why or why not?

[1] If you did not have any collaborators, please say so.

**4. [6 pts]** Consider the problem of deciding whether the equation

$$x^7 + y^7 = z^7 \tag{1}$$

has any solutions where $x, y, z \in \mathcal{N}$. Imagine the following intuitive description of a procedure:

Step 1: For all settings of $x, y, z \in \mathcal{N}$ check whether Equation 1 is satisfied.

Step 2: If some setting during Step 1 had satisfied the equation, *accept*; otherwise, *reject*.

Can a Turing Machine simulate this procedure? Why or why not?

**5. [6 pts]** Let $A$ be a decidable language. Prove that $A^{\leftrightarrow}$ is also decidable. Refer to Problem Set 3 for our definition of $^{\leftrightarrow}$.