MCS 441 – Theory of Computation I
Spring 2014
Problem Set 1

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Due: 2/5/14 at the beginning of class

Instructions: Atop your problem set, please write your name and whether you are an undergraduate or graduate student. Remember: no collaboration is allowed on the problem sets.
Important note: Problems labeled "(**U**)" and "(**G**)" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

1. [**4 pts**] Imagine the following binary sequence: begin with the number 0 and continually append the Boolean negation of the sequence so far. Hence, the sequence begins as

0110100110010110...

One can think of the elements, t_i, of the sequence individually, starting with t_0. So, we have

$$t_0 = 0, t_1 = 1, t_2 = 1, t_3 = 0, t_4 = 1, t_5 = 0, ...$$

What is the value of t_{129}? How did you arrive at your answer?

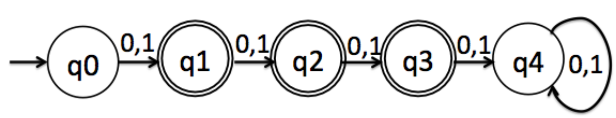
2. [**8 pts**] Draw state diagrams for DFAs recognizing the following languages:

i. $L_1 = \{w \mid \text{length of } w \text{ is even}\}$, $\Sigma = \{1\}$

ii. $L_2 = \{w \mid w \text{ begins with "}bbb\text{" or ends with "}bbb\text{"}\}$, $\Sigma = \{a, b\}$. Restriction: your DFA may contain no more than 8 states.

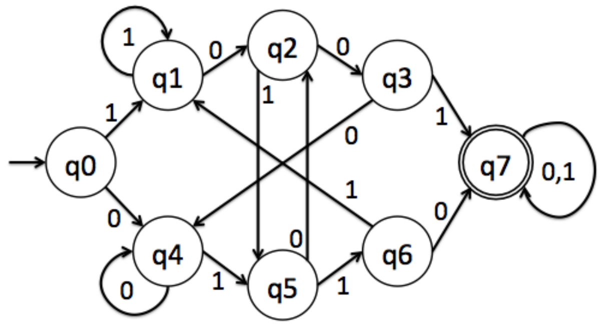
3. [**10 pts**] For each of the following DFAs, explain what language they recognize:

i. (**U**) M_1

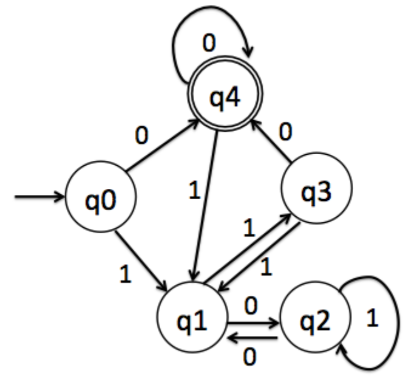


For machine M_1, also give its formal description as a 5-tuple. You do not need to do this for the machines that follow in parts ii. and iii. of this question.

ii. M_2



iii. (**G**) M_3



4. [4 pts] Let A and B be regular languages. Show that $A \backslash B$ is also regular. (Remember that $A \backslash B = \{x | x \in A, x \notin B\}$. Hence, this operation removes all strings from A that are also in B.)

5. [4 pts]
(**U**) Describe all the languages recognizable by 1 state DFAs over $\Sigma = \{0, 1\}$.
(**G**) Give an upper bound on the number of different languages recognizable by an n state machine over an alphabet of size s, as a function of n and s. Explain why your bound is valid.

6. [10 pts] Consider the NFA: $N = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$, with δ defined in Table 1.

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\{q_1\}$	$\{q_1\}$

Table 1: The transition function δ for N

i. [3 pts] Draw the state diagram for N.

ii. [3 pts] What language does N recognize?

iii. [3 pts] Let M_1 be a DFA recognizing $L(N)$. Using the "power set" construction in the proof of Theorem 1.39 from Sipser, draw the state diagram for M_1, labeling the states of M_1 with the corresponding members of $\mathcal{P}(\{q_1, q_2\})$.

iv. [1 pts] Let M_2 be a DFA recognizing $L(M_1)$ but containing fewer states than M_1. Draw the state diagram of M_2.