MCS 441 – Theory of Computation I

Spring 2014
Problem Set 2

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Due: 2/12/14 at the beginning of class

Instructions: Atop your problem set, please write your name and whether you are an undergraduate or graduate student. Remember: no collaboration is allowed on the problem sets.
Important note: Problems labeled "(**U**)" and "(**G**)" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice and are responsible for understanding the answers to those questions.

1. [6 pts] Remember that GNFAs may have only one accept state but can still recognize any regular language.

i. [3 pts] If we allowed NFAs to have only one accept state, would they still be able to recognize any regular language? Why or why not?

ii. [3 pts] How about DFAs? Why or why not?

2. [6 pts] For a string $w = w_1 w_2 \dots w_n$, let $w^{\leftrightarrow} = w_n w_{n-1} \dots w_1$; further, let $\epsilon^{\leftrightarrow} = \epsilon$. For a language A, define the operation

$$A^{\leftrightarrow} = \{w^{\leftrightarrow} \mid w \in A\}.$$

Show that A is regular if and only if $A^{\leftrightarrow}$ is regular.

3. [8 pts] This question explores the conciseness of representation of regular languages.

i. [1 pt] Argue that if a language can be recognized by a DFA with k states then it can also be recognized by an NFA with k states.

Let $\Sigma^n = \underbrace{\Sigma\Sigma\dots\Sigma}_{n}$. Consider the regular language $R_1 = \Sigma^* 1 \Sigma^{k-1}$ over $\Sigma = \{0, 1\}$.

ii. [2 pts] Show that R_1 can be recognized by an NFA with $k + 1$ states.

iii. [4 pts] Prove that any DFA that recognizes R_1 must have at least 2^k states.

You can get full credit for the next questions even if you were not able to answer parts i. – iii.

iv. [1 pt] What does part iii. of this question tell you about Theorem 1.39 from Sipser?

v. [1 pts] What do parts i., ii., and iii. of this question tell you about DFAs as compared to NFAs? Be concrete.

4. [6 pts] Let $\Sigma = \{0, 1\}$. Give regular expressions for the following languages.

a. [3 pts] $\{w \mid \text{every even position of } w \text{ has a } 0\}$

b. [3 pts] $\{w \mid w \text{ contains at least four 1s}\}$

5. [6 pts] Let $\Sigma = \{0, 1\}$. Convert the following regular expressions to NFAs recognizing the same language. Draw the state diagrams for the NFAs.[1]

a. [3 pts] $\Sigma^* 11 \Sigma^*$

b. [3 pts] $((11)^* 00 \cup 01)^*$

6. [6 pts] Use the Pumping Lemma to show the following languages are not regular.

a. [3 pts] $\{www \mid w \in \Sigma^*\}$, $\Sigma = \{0, 1\}$.

b. [3 pts] $\{1^{2^n} \mid n \geq 1\}$, $\Sigma = \{1\}$.

7. (U) [5 pts] Let $\Sigma = \{q, r, s\}$. Consider the language:

$$L = \{q^i r^j s^k \mid i, j, k \geq 0 \text{ and } (i = 1) \rightarrow (j = k)\}.$$

a. [3 pts] Does L satisfy the conditions of the pumping lemma? Why or why not?

b. [2 pts] What does the answer to a. imply about L?

7. (G) [5 pts] Let $\Sigma = \{0, 1\}$. Consider the language:

$$\{1^n x 1^n \mid n \geq 1, x \in \Sigma^*\}.$$

Is the language regular or not? Prove your answer correct.

[1] You may use Lemma 1.55 or solve these some other way.