

MCS 441 – Theory of Computation I
 Spring 2014
 Problem Set 3

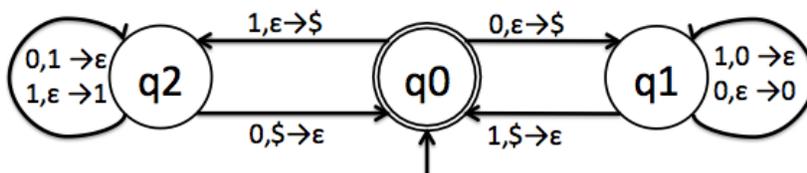
Lev Reyzin

Due: 3/5/14 at the beginning of class

Instructions: Atop your problem set, write your name, and indicate whether you are an undergraduate or graduate student. Answer all questions in the order they are assigned.

Important note: Problems labeled “(U)” and “(G)” are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice and are responsible for understanding the answers to those questions.

1. [10 pts] Let PDA P_1 be:



- a. [2 pts] Give a formal description of P_1 . (Be careful in this part because points will be deducted for small mistakes.)
- b. [3 pts] What language does P_1 recognize? (Give an English description.)
- c. [3 pts] Give a CFG that generates $L(P_1)$.
- d. [2 pts] Is $L(P_1)$ regular? Why or why not?

2. [10 pts] Consider the CFG G_2 :

$$\begin{aligned} S &\rightarrow 0X \\ X &\rightarrow 0X \\ X &\rightarrow 1X \\ X &\rightarrow 1 \end{aligned}$$

- a. [3 pts] What language does G_2 generate?

- b. [2 pts] Is $L(G_2)$ regular? Why or why not?
- c. [3 pts] Give a CFG in CNF generating $L(G_2)$.
- d. [2 pts] Is your CFG in [c.] ambiguous? Why or why not?

3. [5 pts] Consider the following languages over $\Sigma = \{0, 1\}$:

$$L_{3,U} = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$$

$$L_{3,G} = \{st \mid s, t \in \Sigma^*, s \neq t, |s| = |t|\}$$

(U) Is $L_{3,U}$ context free? Show your answer is correct.

(G) Is $L_{3,G}$ context free? Show your answer is correct.¹

4. [5 pts] Prove that if A is a context-free language, then so is A^{\leftrightarrow} . Refer to Problem Set 2 for our definition of \leftrightarrow .

¹Graduate students can complete 3.(U) for partial credit (3 pts) *instead* of doing 3.(G). For graduate students, no credit for 3.(U) will be given if 3.(G) is attempted.