

MCS 441 – Theory of Computation I  
 Spring 2014  
 Problem Set 3

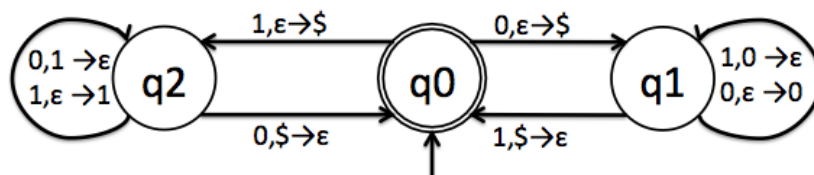
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**Due:** 3/5/14 at the beginning of class

**Instructions:** Atop your problem set, write your name, and indicate whether you are an undergraduate or graduate student. Answer all questions in the order they are assigned.

**Important note:** Problems labeled “(U)” and “(G)” are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice and are responsible for understanding the answers to those questions.

1. [10 pts] Let PDA  $P_1$  be:



- a. [2 pts] Give a formal description of  $P_1$ . (Be careful in this part because points will be deducted for small mistakes.)
- b. [3 pts] What language does  $P_1$  recognize? (Give an English description.)
- c. [3 pts] Give a CFG that generates  $L(P_1)$ .
- d. [2 pts] Is  $L(P_1)$  regular? Why or why not?

2. [10 pts] Consider the CFG  $G_2$ :

$$\begin{aligned}
 S &\rightarrow 0X \\
 X &\rightarrow 0X \\
 X &\rightarrow 1X \\
 X &\rightarrow 1
 \end{aligned}$$

- a. [3 pts] What language does  $G_2$  generate?

- b. [2 pts] Is  $L(G_2)$  regular? Why or why not?
- c. [3 pts] Give a CFG in CNF generating  $L(G_2)$ .
- d. [2 pts] Is your CFG in [c.] ambiguous? Why or why not?

3. [5 pts] Consider the following languages over  $\Sigma = \{0, 1\}$ :

$$L_{3,U} = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$$

$$L_{3,G} = \{st \mid s, t \in \Sigma^*, s \neq t, |s| = |t|\}$$

(U) Is  $L_{3,U}$  context free? Show your answer is correct.

(G) Is  $L_{3,G}$  context free? Show your answer is correct.<sup>1</sup>

4. [5 pts] Prove that if  $A$  is a context-free language, then so is  $A^{\leftrightarrow}$ . Refer to Problem Set 2 for our definition of  $\leftrightarrow$ .

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<sup>1</sup>Graduate students can complete 3.(U) for partial credit (3 pts) *instead* of doing 3.(G). For graduate students, no credit for 3.(U) will be given if 3.(G) is attempted.