MCS 441 – Theory of Computation I

Spring 2014

Problem Set 5

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Due: 4/25/14 at the beginning of class

Instructions: Atop your problem set, write your name, and indicate whether you are an undergraduate or graduate student.

1. [**5 pts**] Prove that if $A, B \in$ P then $A \cup B \in P$.

2. [**5 pts**] Show that the language $K4 \in$ P, where

$$K4 = \{\langle G\rangle |\ G \text{ contains a 4-clique}\}.$$

3. [**6 pts**] Assume that P = NP.

a. Show that this implies that the language

$$\text{RELPRIME} = \{\langle x, y\rangle |\ x \text{ and } y \text{ are relatively prime integers}\}$$

is NP-Complete.

b. Does it imply that every language in NP is NP-Complete? Why or why not?

4. [**4 pts**] Why does definition 7.18 not restrict the length of the certificate c? Informally, what prevents the verifier from cheating and checking a certificate that encodes all possible solutions?

5. [**6 pts**] Consider the problem of making a conference schedule. There are talks $T_1, \ldots, T_k$ to be scheduled and participants $P_1, \ldots, P_\ell$ attending the conference. Each participant gives you a list of the talks he is interested in attending. You must schedule times for these talks so that no participant is interested in two talks that are scheduled for the same time. The problem is to determine if a schedule exists that uses only h slots. Formulate this problem as a language and show it is NP complete.
Hint: reduce from graph 3-COLORING.

6. [**6 pts**] Assume that SAT $\in$ P, where

$$\text{SAT} = \{\langle \phi\rangle |\ \phi \text{ is a satisfiable Boolean formula.}\}$$

Show how to use this fact to find, in polynomial time, an assignment of the variables in ϕ such that ϕ evaluates to *true*.

7. **[8 pts]** Define the class coNP-Complete (analogously to NP-Complete) to be the set of languages A s.t. $A \in$ coNP and $\forall B \in$ coNP, $B \leq_{\mathrm{P}} A$.

a. Let the language

$$\text{ALWAYSTRUE} = \{\langle\phi\rangle | \ \phi \text{ always evaluates to } \textit{true}\}.,$$

e.g. $\phi = (x_1 \vee \overline{x}_1) \in$ ALWAYSTRUE. Prove that ALWAYSTRUE is coNP-Complete.

b. What would be an important consequence of proving ALWAYSTRUE $\in$ NP? Why? (To answer this question, you may assume the statement from part a.)