MCS 590 – Foundations of Data Science  
Spring 2015  
Problem Set 2  
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Due: 3/18/15 at the beginning of class

Instructions: Atop your problem set, please write your name and list your collaborators.

Problems

1. Given the set of integers \{1, 2, \ldots, n\}, what is the expected number of draws \(d\) with replacement so that every integer from the set is drawn? (This is needed for the expected cover time of \(K_n\).)

2. A Markov chain is said to be symmetric if for all \(i\) and \(j\), \(p_{ij} = p_{ji}\). What is the stationary distribution of a connected symmetric chain? Prove your answer.

3. Consider a three state Markov chain with stationary probability \(\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)\), and consider the Metropolis-Hastings algorithm with \(G\) the complete graph on these three vertices. What is the expected probability that a random walk would actually make a move along a selected edge?

4. What is the hitting time \(h_{uv}\) for two adjacent vertices on a cycle of length \(n\)? What is the hitting time if edge \((u, v)\) is removed?

5.\(^1\) Compute the page rank of all the nodes using one of the graphs you used in Problem Set 1, problem 4\(^2\) using restart probability 0.15 (make other reasonable assumptions when needed). Report on your results. How does a vertex’s page rank compare with its degree?

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\(^1\)No collaboration is allowed on this problem.  
\(^2\)If you did not do that problem, you may use a graph of your choosing.