

MCS 590 – Foundations of Data Science
Spring 2015
Problem Set 2

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Due: 3/18/15 at the beginning of class

Instructions: Atop your problem set, please write your name and list your collaborators.

Problems

1. Given the set of integers $\{1, 2, \dots, n\}$, what is the expected number of draws d with replacement so that every integer from the set is drawn? (This is needed for the expected cover time of K_n .)
2. A Markov chain is said to be symmetric if for all i and j , $p_{ij} = p_{ji}$. What is the stationary distribution of a connected symmetric chain? Prove your answer.
3. Consider a three state Markov chain with stationary probability $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$, and consider the Metropolis-Hastings algorithm with G the complete graph on these three vertices. What is the expected probability that a random walk would actually make a move along a selected edge?
4. What is the hitting time h_{uv} for two adjacent vertices on a cycle of length n ? What is the hitting time if edge (u, v) is removed?
- 5.¹ Compute the page rank of all the nodes using one of the graphs you used in Problem Set 1, problem 4,² using restart probability 0.15 (make other reasonable assumptions when needed). Report on your results. How does a vertex's page rank compare with its degree?

¹No collaboration is allowed on this problem.

²If you did not do that problem, you may use a graph of your choosing.