MCS 401 – Computer Algorithms I

Spring 2016

Problem Set 5

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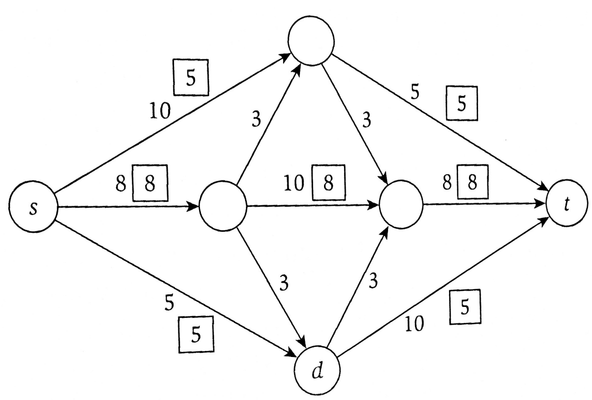
Due: 4/4/16 by the beginning of class

Instructions: Atop your problem set, write your name and whether you are an undergraduate or graduate student. Also write the names of all the students with whom you have collaborated on this problem set.

1. [10 pts] Let G be an arbitrary flow network with a source s, sink t, and positive integer capacity c_e on every edge e. Let (A, B) be a minimum s–t cut of G with respect to these capacities $\{c_e : e \in E\}$. Now, if we add 1 to every capacity, will (A, B) remain a minimum s–t cut with respect to the new capacities $\{1 + c_e : e \in E\}$? Prove your answer correct.

2. [10 pts] Consider the network below in which an s–t flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in the boxes give the amount of flow sent on each edge. (Edges without numbers in boxes have no flow being sent on them.)



What is the value of this flow? Is this a maximum (s, t) flow in the graph? What is the minimum s–t cut in this graph?

3. [10 pts] Let $G = (V, E)$ be a directed graph with a source $s \in V$, sink $t \in V$, and positive integer edge capacities $\{c_e\}$. Give a polynomial time algorithm to decide whether G has *more than one* distinct minimum s–t cut.

4. [10 pts] You are given an undirected graph $G = (V, E)$. A certain collection of nodes $D \subset V$ are designated as *dangerous* and a certain collection of nodes $S \subset V$ is designated as *safe* (assume $D \cap S = \emptyset$). In an emergency, we want a set of evacuation routes from the dangerous nodes to the safe ones. A set of evacuation routes is defined as a set of paths in G such that:

i. each node in D is the tail of one path,

ii. the last node on each path lies in S,

iii. the paths do not share any edges.

Such a collection lets the populations of D escape to S without overly congesting any edge in G.

(a) Given G, D, and S, give a polynomial time algorithm to decide if a set of evacuation routes exists in G.

(b) Suppose we have the same problem as in (a), but we replace condition iii) to say "the paths do not share any *nodes*." Give a polynomial time algorithm for this modified problem.