MCS 441 – Theory of Computation I
Spring 2016
Problem Set 1

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Due: 2/1/16 at the beginning of class

Instructions: Atop your problem set, please write your name and whether you are an undergraduate or graduate student. Please also write the names of all the students with whom you have collaborated on this problem set.
Important note: Problems labeled "(**U**)" and "(**G**)" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

1. [**4 pts**] Imagine the following binary sequence: begin with the number 0 and continually append the Boolean negation of the sequence so far. Hence, the sequence begins as

0110100110010110...

One can think of the elements, t_i, of the sequence individually, starting with t_0. So, we have

$$t_0 = 0, t_1 = 1, t_2 = 1, t_3 = 0, t_4 = 1, t_5 = 0, ...$$

What is the value of t_{129}? How did you arrive at your answer?

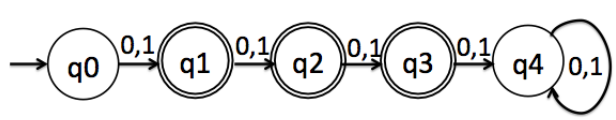
2. [**8 pts**] Draw state diagrams for DFAs recognizing the following languages:

i. $L_1 = \{w \mid \text{length of } w \text{ is even}\}$, $\Sigma = \{1\}$

ii. $L_2 = \{w \mid w \text{ begins with "}bbb\text{" or ends with "}bbb\text{"}\}$, $\Sigma = \{a, b\}$. Restriction: your DFA may contain no more than 8 states.

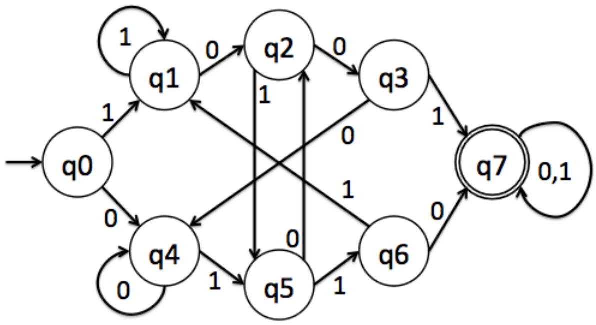
3. [**10 pts**] For each of the following DFAs, explain what language they recognize:

i. (**U**) M_1

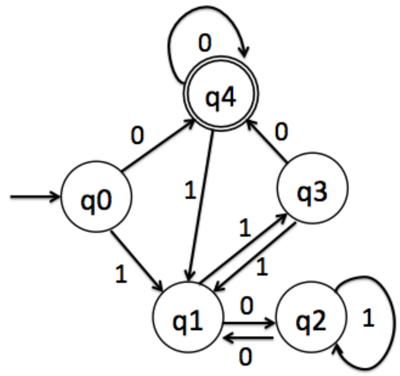


For machine M_1, also give its formal description as a 5-tuple. You do not need to do this for the machines that follow in parts ii. and iii. of this question.

ii. M_2



iii. (**G**) M_3



4. **[4 pts]** Let A and B be regular languages. Show that $A \backslash B$ is also regular. (Remember that $A \backslash B = \{x | x \in A, x \notin B\}$. Hence, this operation removes all strings from A that are also in B.)

5. **[4 pts]**
(**U**) Describe all the languages recognizable by 1 state DFAs over $\Sigma = \{0, 1\}$.
(**G**) Give an upper bound on the number of different languages recognizable by an n state machine over an alphabet of size s, as a function of n and s. Explain why your bound is valid.

6. **[10 pts]** Consider the NFA: $N = (\{q_1, q_2\}, \{0, 1\}, \delta, q_1, \{q_1\})$, with δ defined in Table 1.

δ	0	1	ϵ
q_1	$\{q_1, q_2\}$	$\emptyset$	$\emptyset$
q_2	$\emptyset$	$\{q_1\}$	$\{q_1\}$

Table 1: The transition function δ for N

i. [3 pts] Draw the state diagram for N.

ii. [3 pts] What language does N recognize?

iii. [3 pts] Let M_1 be a DFA recognizing $L(N)$. Using the "power set" construction in the proof of Theorem 1.39 from Sipser, draw the state diagram for M_1, labeling the states of M_1 with the corresponding members of $\mathcal{P}(\{q_1, q_2\})$.

iv. [1 pts] Let M_2 be a DFA recognizing $L(M_1)$ but containing fewer states than M_1. Draw the state diagram of M_2.