# MCS 441 – Theory of Computation I
## Spring 2016
## Problem Set 2

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**Due**: 2/12/16 at the beginning of class

**Instructions:** Atop your problem set, please write your name and whether you are an undergraduate or graduate student. Please also write the names of all the students with whom you have collaborated on this problem set.
**Important note:** Problems labeled "(**U**)" and "(**G**)" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

**1. [6 pts]** Remember that GNFAs may have only one accept state but can still recognize any regular language.

i. [3 pts] If we allowed NFAs to have only one accept state, would they still be able to recognize any regular language? Why or why not?

ii. [3 pts] How about DFAs? Why or why not?

**2. [6 pts]** For a string $w = w_1 w_2 \dots w_n$, let $w^{\leftrightarrow} = w_n w_{n-1} \dots w_1$; further, let $\epsilon^{\leftrightarrow} = \epsilon$. For a language $A$, define the operation

$$A^{\leftrightarrow} = \{w^{\leftrightarrow} \mid w \in A\}.$$

Show that $A$ is regular if and only if $A^{\leftrightarrow}$ is regular.

**3. [8 pts]** This question explores the conciseness of representation of regular languages.

i. [1 pt] Argue that if a language can be recognized by a DFA with $k$ states then it can also be recognized by an NFA with $k$ states.

Let $\Sigma^n = \underbrace{\Sigma\Sigma\dots\Sigma}_{n}$. Consider the regular language $R_1 = \Sigma^* 1 \Sigma^{k-1}$ over $\Sigma = \{0, 1\}$.

ii. [2 pts] Show that $R_1$ can be recognized by an NFA with $k+1$ states.

iii. [4 pts] Prove that any DFA that recognizes $R_1$ must have at least $2^k$ states.

You can get full credit for the next questions even if you were not able to answer parts i. – iii.

iv. [1 pt] What does part iii. of this question tell you about Theorem 1.39 from Sipser?

v. [1 pts] What do parts i., ii., and iii. of this question tell you about DFAs as compared to NFAs? Be concrete.

**4. [6 pts]** Let $\Sigma = \{0, 1\}$. Give regular expressions for the following languages.

a. [3 pts] $\{w \mid \text{every even position of } w \text{ has a } 0\}$

b. [3 pts] $\{w \mid w \text{ contains at least four 1s}\}$

**5. [6 pts]** Let $\Sigma = \{0, 1\}$. Convert the following regular expressions to NFAs recognizing the same language. Draw the state diagrams for the NFAs.[1]

a. [3 pts] $\Sigma^*11\Sigma^*$

b. [3 pts] $((11)^*00 \cup 01)^*$

**6. [6 pts]** Use the Pumping Lemma to show the following languages are not regular.

a. [3 pts] $\{www \mid w \in \Sigma^*\}$, $\Sigma = \{0, 1\}$.

b. [3 pts] $\{1^{2^n} \mid n \geq 1\}$, $\Sigma = \{1\}$.

**7. (U) [5 pts]** Let $\Sigma = \{q, r, s\}$. Consider the language:

$$L = \{q^i r^j s^k \mid i, j, k \geq 0 \text{ and } (i = 1) \rightarrow (j = k)\}.$$

a. [3 pts] Does $L$ satisfy the conditions of the pumping lemma? Why or why not?

b. [2 pts] What does the answer to a. imply about $L$?

**7. (G) [5 pts]** Let $\Sigma = \{0, 1\}$. Consider the language:

$$\{1^n x 1^n \mid n \geq 1, x \in \Sigma^*\}.$$

Is it regular or not? Prove your answer correct.

[1] You may use Lemma 1.55 or solve these some other way.