

MCS 441 – Theory of Computation I
 Spring 2018
 Problem Set 3

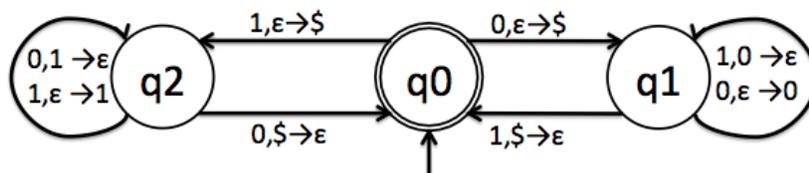
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Due: 3/5/18 at the beginning of class

Instructions: Atop your problem set, please write your name and whether you are an undergraduate or graduate student. Please also write the names of all the students with whom you have collaborated on this problem set.

Important note: Problems labeled “(U)” and “(G)” are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

1. [10 pts] Let PDA P_1 be:



- a. [2 pts] Give a formal description of P_1 . (Be careful in this part because points will be deducted for small mistakes.)
- b. [3 pts] What language does P_1 recognize? (Give an English description.)
- c. [3 pts] Give a CFG that generates $L(P_1)$.
- d. [2 pts] Is $L(P_1)$ regular? Why or why not?

2. [10 pts] Consider the CFG G_2 :

$$\begin{aligned} S &\rightarrow 0X \\ X &\rightarrow 0X \\ X &\rightarrow 1X \\ X &\rightarrow 1 \end{aligned}$$

- a. [3 pts] What language does G_2 generate?

- b. [2 pts] Is $L(G_2)$ regular? Why or why not?
- c. [3 pts] Give a CFG in CNF generating $L(G_2)$.
- d. [2 pts] Is your CFG in [c.] ambiguous? Why or why not?

3. [5 pts] Consider the following languages over $\Sigma = \{0, 1\}$:

$$L_{3,U} = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$$

$$L_{3,G} = \{st \mid s, t \in \Sigma^*, s \neq t, |s| = |t|\}$$

(U) Is $L_{3,U}$ context free? Show your answer is correct.

(G) Is $L_{3,G}$ context free? Show your answer is correct.¹

4. [5 pts] Prove that if A is a context-free language, then so is A^{\leftrightarrow} . Refer to Problem Set 2 for our definition of \leftrightarrow .

¹Graduate students can complete 3.(U) for partial credit (3 pts) *instead* of doing 3.(G). For graduate students, no credit for 3.(U) will be given if 3.(G) is attempted.