Final[1], Due 11:59pm, 5/6 ECON/STAT 473, Game Theory, Spring 2020

During this exam, you *are allowed* to use the course textbook, course notes, course lectures, and anything posted on the course Piazza site. In your answers, you may refer to any result proved in class, the book, or homework. **But you *may not* collaborate on answers, use any outside resources, or search for hints online.** Good luck!

Problem 1. (25 points) Consider the two-player game where player 1 chooses *odd* or *even* and player 2 simultaneously chooses the number 3 or 8. If player 1 guesses the parity of player 2's number, player 2 pays player 1 that amount, but if player 1 guesses wrong, then player 1 owes player 2 that amount. E.g., if player 1 says *odd* and player 2 chooses 8, then player 1 owes \$8 to player 2.

Give the payoff matrix that represents this two-player zero-sum game. What are optimal strategies for players 1 and 2? What is the value of this game?

Problem 2. (30 points) Consider the game "Golden Balls" where two players must divide a pot of money by each choosing *split* or *steal*. If both players choose *split*, the pot is divided evenly. If one chooses *split* and the other chooses *steal*, the player who chose *steal* gets the entire pot. If both choose *steal*, both get nothing.

In class, we noted that *steal* is a dominant strategy and (*steal*,*steal*) is a Nash equilibrium. However, we might ask whether there exists a correlated equilibrium that gives a non-zero expected payoff to each player. Does such an equilibrium exist? If so, what is one such equilibrium and why? If not, why not?

Problem 3. (25 points) Consider the following simplified game between the USSR (player 1) and the USA (player 2). First, the USSR needs to decide whether to launch a nuclear attack on the USA or not. If the USSR decides not to attack, then the payoffs are $(0, 0)$. If the USSR does attack, the USA needs to decide whether to retaliate with a nuclear strike of its own. If the USA does not retaliate, then the payoffs are $(100, -10000)$. If the USA does retaliate, the payoffs are $(-M, -M)$, where M is a huge number ($M >> 10000$) whose negation corresponds to the end of life on Earth.

Write the game tree of this extensive-form game. Use backwards induction to find the resulting outcome if both players play according to their best strategies. Open-ended: what could the USA do to potentially change the game to avoid this bad outcome?

Problem 4. (30 points) Consider a game where two players simultaneously choose A or B and both get a payoff of 1 if they choose the same letter and both get a payoff of 0 if they choose different letters. Is both players playing $(1/2, 1/2)$ a Nash equilibrium? If so, is $(1/2, 1/2)$ also an evolutionarily stable equilibrium? Why or why not?

Now consider a similar game where where two players simultaneously choose A or B and both get a payoff of 1 if they choose different letters and both get a payoff of 0 if they choose the same letter. Is both players playing $(1/2, 1/2)$ a Nash equilibrium? If so, is $(1/2, 1/2)$ also an evolutionarily stable equilibrium? Why or why not?

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[1]A raw score of 80+ will be an A, 60+ *at least* a B, 40+ *at least* a C, and 20+ *at least* a D. The total number of possible points is 110, so you may think of 10 of these points as "extra credit."