Some solutions for the math 125 final review packet

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The students in my section decided via democratic process the problems in the Math 125 final exam practice packet for which they wanted to see solutions. I this guide I do not write out the statements of the problems, so one will need a copy of the Math 125 final review (Click in the final review button at the Math 125 Exams page.) I have no doubt that this document contains typos or mistakes; please let me of these. Good luck, and study well.

4. Mine 1 produces 4 tons of iron and 1 ton of copper at \$3,200 a day, and mine 2 produces 2 tons of iron and 2 tons of copper at \$2,600 a day. We need at least 40 tons of iron and 18 tons of copper. Let x be the number of days running mine 1 and y be the number of days running mine 2. Then we have the following system.

$$\begin{cases} \text{Minimize } 3,200x + 2,600y \\ 4x + 2y \geq 40 \\ 2x + 2y \geq 18 \\ x, y \geq 0 \end{cases}$$

6.

- $$\begin{pmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 1 & -2 & -3 & -7 & 5 \end{pmatrix} (R_4 \to 4R_4) \to \begin{pmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 4 & -8 & -12 & -28 & 20 \end{pmatrix}$$

- $$\begin{pmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 1 & -2 & -3 & -7 & 5 \end{pmatrix} (R_1 \to -2R_4 + R_1) \to \begin{pmatrix} 1 & 6 & 4 & 8 & -5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 1 & -2 & -3 & -7 & 5 \end{pmatrix}$$

- $$\begin{pmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 0 & -4 & 5 & 1 & 3 \\ 1 & -2 & -3 & -7 & 5 \end{pmatrix} (R_3 \to 5R_1 + R_3) \to \begin{pmatrix} 3 & 2 & -2 & -6 & 5 \\ -3 & 12 & 7 & -2 & -6 \\ 15 & 6 & -5 & -29 & 28 \\ 1 & -2 & -3 & -7 & 5 \end{pmatrix}$$

11.

	Metal	*Plastic*	*Wood*
Metal	0.08	0	0.05
Plastic	0.03	0.11	0.04
Wood	0.07	0.02	0.10

Thus, we have that

$$A = \begin{pmatrix} 0.08 & 0 & 0.05 \\ 0.03 & 0.11 & 0.04 \\ 0.07 & 0.02 & 0.10 \end{pmatrix}$$

We are given that $D = \begin{pmatrix} 32 \\ 25 \\ 40 \end{pmatrix}$ where the numbers in D denote millions of dollars. Now on your calculator, compute $(I - A)^{-1}D$. Round this to the hundredths place. Remember that you must multiply with D on the right.

13. Let x be the number of package A, and y be the number of package B. From the statement of the problem we see that or linear programming question is the following.

$$\begin{cases} \text{Maximize } 175x + 240y \\ 140x + 185y \leq 6,000 \\ 3x + 5y \leq 300 \\ x, y \geq 0 \end{cases}$$

14. Let x be the number of tablets and y be the total cost. This question gives us two linear equations. For the Thailand plant we have $y = 79x + 9,000$ and for the Malaysia plant we have $y = 71x + 9,400$.

- This question is just asking us to set $x = 600$ and see which function has a lesser value. In particular the costs at Thailand are $\$79 \cdot 600 + \$9,000 = \$56,400$ and similarly the costs at Malaysia are $\$52,000$. Thus, it's cheaper to produce in Malaysia.
- Let x_1 be the number of units made in Thailand and x_2 be the number of units made in Malaysia. Since we have to produce 1000 units we see that $x_1 = 1000 - x_2$. All that is left to do is set the two linear equations to be equal and plug in $1000 - x_2$ for x_1 in the Thailand equation. Solving $79(1000 - x_2) + 9,000 = 71x_2 + 9,400$ for x_2 we have that $x_2 = 524$ (524 tablets must be made in Malaysia), and thus $x_1 = 476$ (476 tablets must be made in Thailand).

15.

- We use the data given to set up the following table.

	Millions of people	Number of stoves sold
Atlanta	6.1	13,286
Tampa	2.8	5,123
Miami	6.4	17,522
Charlotte	2.5	4,848
Greenville	1.4	3,613

  Using the LinReg feature we get that the line of best fit is $y = 2647x - 1286$. [1]

- Just plug in $x = 2.9$ to the equation we just found.
- Solving $20,000 = 2647x - 1286$ for x gives us that we need about $x = 8.04$ million people.

[1](The problem says to round to the nearest thousandth, but I don't have a TI calculator right now, this is the idea, and this will be very close to the answer you will get with your calculator.)

17. We need a linear programming problem (LPP) in two variables, but at first lets start with a LPP in three. Let L be the number of lions, B be the number of bears, and T be the number of tigers. Reading the question carefully will give us the following LPP in three variables.

$$\begin{cases} \text{Minimize 40L+30T+25B} \\ L+B+T=150 \\ 2L \geq T \\ B \leq T \\ B, L, T \geq 25 \end{cases}$$

However, we notice that we can solve the equation $L+B+T=150$ for one of the variables. You may pick you favorite one, but I'm solving the question right now and my favorite one is L. So we have $L=150-B-T$, and we can substitute this L in the LPP to reduce the number of variables.

$$\begin{cases} \text{Minimize 40(150-B-T)+30T+25B} \\ (150-B-T)+B+T=150 \\ 2(150-B-T) \geq T \\ B \leq T \\ B, (150-B-T), T \geq 25 \end{cases}$$

This is a mess. Note the the second equality reduces to 150=150, so we discard that equality now. Also the last row gives is that $(150-B-T) \geq 25 \Rightarrow B+T \leq 125$ which is a non-trival inequality. We need to include in in our new LPP. Now just clean up the rest of the inequalities to get

$$\begin{cases} \text{Minimize 6000-15B-10T} \\ B+T \leq 125 \\ 2B+3T \leq 300 \\ B-T \leq 0 \\ B, T \geq 25 \end{cases}$$

19.

- The first constraint is changing from 34 to 30, so in this case $h=-4=30-34$. As written, the first constraint corresponds to the u column, so the new right hand column is

$$\begin{pmatrix} 2-4(-5/2) \\ 4-4(1) \\ 13-4(-1/2) \\ 47-4(1/2) \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 15 \\ 45 \end{pmatrix}$$

- Mostly the same story. 30 is changing to 32 so $h=2=32-30$. Also, the third constraint corresponds to the w column so the new right hand side is

$$\begin{pmatrix} 2+2(1) \\ 4+2(-1) \\ 13+2(1) \\ 47+2(1) \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 15 \\ 49 \end{pmatrix}$$

- In this case, we have that 34 is changing to 34 + h, as above we just look at the difference of $34+h-34 = h$. Constraint 1 corresponds to the u column so we have

$$\begin{pmatrix} 2+h(-5/2) \\ 4+h(1) \\ 13+h(-1/2) \\ 47+h(1/2) \end{pmatrix}$$

since the M value doesn't matter when we are looking to remain feasible, we only need to look at the following inequalities.

$$\begin{cases} 2+h(-5/2) \geq 0 \\ 4+h(1) \geq 0 \\ 13+h(-1/2) \geq 0 \end{cases}$$

Clean this up to get

$$\begin{cases} h \leq 4/5 \\ h \geq -4 \\ h \leq 26 \end{cases}$$

To make this even clearer, rewrite this as

$$\begin{cases} h \leq 4/5 \\ -4 \leq h \\ h \leq 26 \end{cases}$$

We can now see that $-4 \leq h \leq 4/5$ if we want to stay feasible.

25. We are given that $AB = C$ and asked to find $b_{2,1} + b_{2,2}$. As we discussed in the discussion sections this just means we have to find the entries in the second row first column and the second row second column in B and add them together. Since we know A and C and we see that the determinant of A is not 0, we know that we can invert A. This gives us that

$$B = A^{-1}C$$

Using either your calculator or doing the calculation by hand, we will see that $A^{-1} = \begin{pmatrix} 2 & 1 \\ 5/2 & 3/2 \end{pmatrix}$ Thus

$$B = A^{-1}C = \begin{pmatrix} 2 & 1 \\ 5/2 & 3/2 \end{pmatrix} \begin{pmatrix} -6 & -2 \\ 11 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3/2 & 4 \end{pmatrix}$$

The entry in the second row first column is 3/2 and the entry in the second row second column is 4. So $b_{2,1} + b_{2,2} = 3/2 + 4 = 11/2$

29. Let's define the payoff matrix as the money that Player C must pay Player R. The matrix then looks like this

		Player R		
		$1	$10	$50
Player C	$5	4	−5	−45
	$20	19	10	−40

Asking if this game is strictly determined is asking if this matrix has saddle point. So we need to perform the following.

1) Determine the least element in each row. (-40,-45)
2) Choose the largest of these elements. (-40)
3) Determine the greatest element in each column. (19, 10, -40)
4) Choose the smallest of these elements. (-40)

If the element chosen in step 2 and 4 is the same (it is) we have found our saddle point.

Following this guide we see that -40 is the saddle point. Therefore, thus game is strictly determined, and the Value of the game is $v = -40$

31. (With David N. Reynolds) We start with the matrix $\begin{pmatrix} 0 & -3 \\ -1 & 2 \end{pmatrix}$. We need to add 4 to make every entry positive. So we have the matrix $\begin{pmatrix} 4 & 1 \\ 3 & 6 \end{pmatrix}$ We now construct the LPP for the optimal strategy for player C.

$$\text{Maximize } z_1 + z_2$$

$$\begin{cases} 4z_1 + z_2 \leq 1 \\ 3z_1 + 6z_2 \leq 1 \\ z_1, z_2 \geq 0 \end{cases}$$

Which gives us the following initial tableau.

$$\left[\begin{array}{ccccc|c} z_1 & z_2 & u_1 & u_2 & M & \\ 4 & 1 & 1 & 0 & 0 & 1 \\ 3 & 6 & 0 & 1.4 & 0 & 1 \\ \hline -1 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

Which will give us the following final tableau.

$$\left[\begin{array}{ccccc|c} z_1 & z_2 & u_1 & u_2 & M & \\ 1 & 0 & 2/7 & -1/21 & 0 & 5/21 \\ 0 & 1 & -1/7 & 4/21 & 0 & 1/21 \\ 0 & 0 & 1/7 & 1/7 & 1 & 2/7 \end{array}\right]$$

Note that this gives us that $v = \frac{1}{z_1+z_2} = \frac{1}{5/21+1/21} = \frac{21}{6} = \frac{7}{2}$. Now do the following computations.

- $z_1 \cdot v = \frac{5}{21} \cdot \frac{7}{2} = \frac{5}{6}$
- $z_2 \cdot v = \frac{1}{21} \cdot \frac{7}{2} = \frac{1}{6}$

  These first two computations give us the entries for the optimal mixed strategy for C, while these next two give us the entries for the optimal mixed strategy for R.

- $u_1 \cdot v = \frac{1}{7} \cdot \frac{7}{2} = \frac{1}{2}$
- $u_2 \cdot v = \frac{1}{7} \cdot \frac{7}{2} = \frac{1}{2}$

So the optimal mixed strategy for C is $\begin{bmatrix} \frac{5}{6} \\ \frac{1}{6} \end{bmatrix}$ and the optimal mixed strategy for R is $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ The last step is to remember that the v we found it the value for the matrix we shifted by 4. Thus, the value of the original matrix is $v - 4 = \frac{7}{2} - 4 = -\frac{1}{2}$.