Math 180 Practice Exam 1

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Name

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1) Compute the following limits.

- $\lim_{x\to -5} \frac{x^2-8x-65}{x^2+6x+5}$

$$\begin{aligned}\lim_{x\to -5} \frac{x^2-8x-65}{x^2+6x+5} &= \lim_{x\to -5} \frac{(x-13)(x+5)}{(x+1)(x+5)} \\ &= \lim_{x\to -5} \frac{(x-13)}{(x+1)} \\ &= \frac{9}{2}\end{aligned}$$

- $\lim_{x\to 0} \frac{\tan(x)(x^2-3x+2)}{x^2-x}$

$$\begin{aligned}\lim_{x\to 0} \frac{\tan(x)(x^2-3x+2)}{x^2-x} &= \lim_{x\to 0} \frac{\sin(x)}{x} \frac{(x^2-3x+2)}{\cos(x)\cdot(x-1)} \\ &= \lim_{x\to 0} \frac{\sin(x)}{x} \frac{(x-1)(x-2)}{\cos(x)\cdot(x-1)} \\ &= \lim_{x\to 0} \frac{\sin(x)}{x} \frac{(x-2)}{\cos(x)} \\ &= 1\cdot\frac{-2}{1} \\ &= -2\end{aligned}$$

- $\lim_{x\to\infty} \frac{x^7}{1{,}000{,}000{,}000{,}000\cdot x^4+3{,}000\cdot x^4+x}$

  The degree of the numerator is greater than the degree of the denominator. Therefore

$$\lim_{x\to\infty} \frac{x^7}{1,000,000,000,000\cdot x^4+3,000\cdot x^4+x} = \infty$$

- $\lim_{x\to 3} \frac{1}{6-\sqrt{2x}}$

$$\begin{aligned}\lim_{x\to 3} \frac{1}{6-\sqrt{2x}} &= \lim_{x\to 4} \frac{1}{6-\sqrt{2x}}\cdot\frac{6+\sqrt{2x}}{6+\sqrt{2x}} \\ &= \lim_{x\to 3} \frac{6+\sqrt{2x}}{(6+\sqrt{x})(6-\sqrt{2x})} \\ &= \lim_{x\to 3} \frac{6+\sqrt{2x}}{(6+\sqrt{2x})(6-\sqrt{2x})} \\ &= \lim_{x\to 3} \frac{6+\sqrt{2x}}{(36-2x)} \\ &= \frac{6+\sqrt{6}}{30}\end{aligned}$$

2) Compute the following derivatives.

- $\frac{d}{dx}\left(x \cdot \tan(x)\right)$
  $\frac{d}{dx}\left(x \cdot \tan(x)\right) = \tan(x) + x \cdot \sec^2(x)$

- $\frac{d}{dx}\left(3x^5 + 4x^2 + e^{-2x}\right)$
  $\frac{d}{dx}\left(3x^5 + 4x^2 + e^{-2x}\right) = 15x^4 + 8x - 2e^{-2x}$

- $\frac{d}{dx}\left(x^2 e^x\right)$
  $\frac{d}{dx}\left(x^2 e^x\right) = 2xe^x + x^2 e^x$

- $\frac{d}{dx}\left(\sin(5xe^x)\right)$
  $\frac{d}{dx}\left(\sin(5xe^x)\right) = \cos(5xe^x)\left(5e^x + 5xe^x\right)$

- $\frac{d}{dx}\left(\frac{x^3}{e^x}\right)$
  $\frac{d}{dx}\left(\frac{x^3}{e^x}\right) = \frac{x^3 e^x - 3x^2 e^x}{e^2 x}$

3) Using the Intermediate Value Theorem, prove the following.

$f(x) = 2x^3 - x^2 + 2x - 1$ has a root in the interval $(0, 1)$.

Note that $f(x)$ is a polynomial, so it is continuous on the whole real line. Now note that $f(0) = -1$ and $f(1) = 2$ and $-1 < 0 < 2$. So, by the intermediate value theorem there exists c in the interval $(0, 1)$ so that $f(c) = 0$. By definition, c is a root (or zero) of $f(x)$.

4) Sketch a function $f(x)$ which has the following properties.

- $f(x) \geq 0$ for $x \leq 0$
- $\lim_{x \to 0^+} f(x) = 0$
- $\lim_{x \to 0^-} f(x) = 4$
- $\lim_{x \to \infty} f(x) = -2$
- $\lim_{x \to -\infty} f(x) = 0$

5) Find $\lim_{x\to 0} x \cdot \sin(\frac{1}{x})$. Name any theorems used.

Since $-|x| \leq x \leq |x|$ and $-1 \leq \sin(\frac{1}{x}) \leq 1$ we have that

$$|x| \leq x \cdot \sin(\frac{1}{x}) \leq |x|$$

Since $\lim_{x\to 0} -|x| = 0 = \lim_{x\to 0} |x|$ we may conclude by the squeeze theorem that

$$\lim_{x\to 0} x \cdot \sin(\frac{1}{x}) = 0$$

6) A ball is thrown into that air and falls to he ground. The height (in meters) is a function of t (in seconds) given as

$$h(t) = -4.9t^2 + 98t + 1.4$$

a) What was the initial velocity of the ball?

To find the velocity, we take the derivative of the position function $h(t)$. We can see by the power rule that $h'(t) = -9.8t + 98$, so $h'(0) = 98$ m/sec

b) What was the height the ball was released from?

$h(0) = 1.4$ meters

c) What is the acceleration of the ball?

We take the second derivative and find that $h''(t) = -9.8$ m/sec^2 (Plus Ultra!!!)

d) When is the ball at it's highest point?

The ball is at it's highest point when the velocity changes from positive to negative. That is when $v(t) = 0$. So we set $v(t) = -9.8t + 98 = 0$ and solve for t. We find that $t = 10$ seconds.