Quiz 1

1. (Exercise 24, §13.5) Find an equation for the plane Π through the point \( P = (4, 0, -3) \) with normal vector \( \mathbf{n} = j + 2k \).

   **Answer:** Let \( \mathbf{r}_0 = \langle 4, 0, -3 \rangle \) and let \( (x, y, z) \) be in \( \Pi \). \( (x, y, z) \) corresponds with the vector \( \mathbf{r} = \langle x, y, z \rangle \) and we would therefore have that \( \mathbf{r} - \mathbf{r}_0 \) is “contained” in \( \Pi \). Then \( \mathbf{r} - \mathbf{r}_0 \) would be orthogonal to \( \mathbf{n} \) so that

   \[
   0 = (\mathbf{r} - \mathbf{r}_0) \cdot \mathbf{n} = (\langle x, y, z \rangle - \langle 4, 0, -3 \rangle) \cdot (0, 1, 2) = \langle x - 4, y, z + 3 \rangle \cdot (0, 1, 2) = y + 2z + 6.
   \]

2. (Exercise 15, §13.5) Find symmetric equations for the line \( \ell \) that passes through the point \((0, 2, -1)\) and is parallel to the line given by

   \[
   \begin{aligned}
   x &= 1 + 2t \\
   y &= 3t \\
   z &= 5 - 7t
   \end{aligned}
   \]

   **Answer:** The line given in (1) has vector equation

   \[
   \mathbf{r}_0 + t\mathbf{v}
   \]

   where \( \mathbf{r}_0 = \langle 1, 0, 5 \rangle \) and \( \mathbf{v} = \langle 2, 3, -7 \rangle \). (Verify this!) Now since \( \ell \) is parallel to this line, \( \ell \) must have the same direction part \( \mathbf{v} \). We can then take \( \ell \) to be given by the vector equations

   \[
   \mathbf{r}_1 + t\mathbf{v}
   \]

   where \( \mathbf{r}_1 = \langle 0, 2, -1 \rangle \). In coordinates, \( \ell \) is given by

   \[
   \begin{aligned}
   \langle 2t, 2 + 3t, -1 - 7t \rangle
   \end{aligned}
   \]

   or

   \[
   \begin{aligned}
   x &= 2t \\
   y &= 2 + 3t \\
   z &= -1 - 7t
   \end{aligned}
   \]

   so that the symmetric equations for \( \ell \) are

   \[
   \frac{x}{2} = \frac{y - 2}{3} = \frac{z + 1}{-7}.
   \]

   \( \ell \) intersects the \( xy \)-plane at the point where \( z = 0 \). Solving the third equation in (2) for \( t \), we have

   \[
   0 = z = -1 - 7t \quad \implies \quad t = \frac{-1}{7}.
   \]

   Substituting the value \( t = \frac{-1}{7} \) into the other equations in (2), we have \( x = \frac{-2}{7} \) and \( y = \frac{11}{7} \). This means that \( \ell \) intersects the \( xy \)-plane at the point \( \left( \frac{-2}{7}, \frac{11}{7}, 0 \right) \). Similarly, \( \ell \) intersects the \( yz \)-plane at \( \left( \frac{-4}{3}, 0, \frac{11}{3} \right) \) and intersects the \( yz \)-plane at \( (0, 2, -1) \).