Quiz 1

1. Find two unit vectors which are orthogonal to both \( u = i + j + k \) and \( v = 2i + k \).

   **Answer:** The vectors
   
   \[
   u \times v = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = i + j - 2k
   \]
   
   and
   
   \[
   v \times u = -(u \times v) = -i - j + 2k
   \]

   are both orthogonal to both \( u \) and \( v \). Normalizing, we have that
   
   \[
   \frac{i + j - 2k}{|i + j - 2k|} = \frac{i + j - 2k}{\sqrt{6}}
   \]

   and
   
   \[
   \frac{-i - j + 2k}{|-i - j + 2k|} = \frac{-i - j + 2k}{\sqrt{6}}
   \]

   are unit vectors which are orthogonal to both \( u \) and \( v \).

2. Recall that the angle between two intersecting planes is the angle between the vectors which are normal to the planes. Two intersecting planes are perpendicular if the angle between them is \( \pi/2 \). Determine whether the planes \( \pi_1 \) given by \( x + y + z = 1 \) and \( \pi_2 \) given by \( x - y + z = 1 \) are parallel, perpendicular, or neither. If neither, find the angle between them.

   **Answer:** The vector \( n_1 = \langle 1, 1, 1 \rangle \) is normal to \( \pi_1 \) and the vector \( n_2 = \langle 1, -1, 1 \rangle \) is normal to \( \pi_2 \). If \( \theta \) is the angle between \( n_1 \) and \( n_2 \), then
   
   \[
   \cos \theta = \frac{u \cdot v}{|u||v|} = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}
   \]

3. Find symmetric equations for the line \( \ell \) that passes through the points \((6, 1, -3)\) and \((2, 4, 5)\).

   **Answer:** The vector \( v = \langle 6 - 2, 1 - 4, -3 - 5 \rangle = \langle 4, -3, -8 \rangle \)

   goes from \((6, 1, -3)\) to \((2, 4, 5)\), so \( \ell \) should have direction vector \( \ell \). Then \( \ell \) is given either by
   
   \[
   \frac{x - 6}{4} = \frac{y - 1}{-3} = \frac{z + 3}{-8}
   \]

   or
   
   \[
   \frac{x - 2}{4} = \frac{y - 4}{-3} = \frac{z - 5}{-8}
   \]