1. The length of the arc of \( \mathbf{r}(t) \) between \( \mathbf{r}(a) \) and \( \mathbf{r}(b) \) is given by
\[
\int_a^b |\mathbf{r}'(t)|\,dt.
\]

2. The curvature of the curve \( \mathbf{r}(t) \) is given by
\[
\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.
\]

3. If \( z \) is a function of \( x \) and \( y \), which in turn are functions of \( u \) and \( v \), then
\[
\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}, \quad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}.
\]

4. The directional derivative of \( f(x, y, z) \) in the direction of the unit vector \( \mathbf{u} \) is given by
\[
\nabla \cdot \mathbf{u} = \nabla f(x, y, z) \cdot \mathbf{u}.
\]

5. The area of the part of the surface \( z = f(x, y) \) which is the image under \( f \) of the region \( D \subset \mathbb{R}^2 \) is given by
\[
\int_D \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1} \,dA.
\]

6. The change of coordinates from spherical to rectangular is given by
\[
x = \rho \sin \varphi \cos \theta \\
y = \rho \sin \varphi \sin \theta \\
z = \rho \cos \varphi.
\]

7. Let \( E \) be the spherical wedge \( \{(\rho, \theta, \varphi) : a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \varphi \leq d\} \). Then
\[
\iiint_E f(x, y, z) \,dV = \int_c^d \int_\alpha^\beta \int_a^b f\left(\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi\right) \rho^2 \sin \varphi \,d\rho \,d\theta \,d\varphi.
\]

8. The Jacobian of the transformation \( T(u, v) = (x(u, v), y(u, v)) \) is defined
\[
\frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}
\]

9. \[
\iint_{T(S)} f(x, y) \,dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \,dA.
\]
10. Let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field and let the curve $C$ be the image of $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P \, dx + Q \, dy = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt.$$ 

11. Let $C$ be a positively oriented, piecewise-smooth, simple, closed curve in the plane and let $D$ be the region bounded by $C$. Then

$$\int_C P \, dx + Q \, dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA.$$