

Worksheet 1 Answers

1.

$$\begin{aligned}\int_0^3 \int_{y^2}^9 y \sin(x^2) dx dy &= \int_0^9 \int_0^{\sqrt{y}} y \sin(x^2) dy dx = \int_0^9 \left[\frac{1}{2} y^2 \sin(x^2) \right]_0^{\sqrt{y}} dx \\ &= \int_0^9 \frac{1}{2} x \sin(x^2) dx = -\frac{1}{4} [\cos x^2]_0^9 = -\frac{1}{4} [\cos 81 - 1]\end{aligned}$$

2.

$$\begin{aligned}\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3} dx dy &= \int_0^1 \int_0^{x^2} y \sqrt{2+x^3} dy dx = \int_0^1 \left[y \sqrt{2+x^3} \right]_0^{x^2} dx \\ &= \int_0^1 x^2 \sqrt{2+x^3} dx = \frac{2}{3} \cdot \frac{1}{3} (2+x^3)^{\frac{3}{2}} \Big|_0^1 = \frac{2}{9} [3^{\frac{3}{2}} - 2^{\frac{3}{2}}]\end{aligned}$$

3.

$$\begin{aligned}\int_0^1 \int_y^1 \sin x^2 dx dy &= \int_0^1 \int_0^x \sin x^2 dy dx = \int_0^1 y \sin x^2 \Big|_0^x dx \\ &= \int_0^1 x \sin x^2 dx = -\frac{1}{2} \cos x^2 \Big|_0^1 = \frac{1}{2} (1 - \cos 1)\end{aligned}$$

4.

$$\int_0^1 \int_{e^y}^e \frac{x}{\ln x} dx dy = \int_1^e \int_0^{\ln x} \frac{x}{\ln x} dy dx = \int_1^e \frac{xy}{\ln x} \Big|_0^{\ln x} dx = \int_1^e x dx = \frac{1}{2} (e^2 - 1)$$