Decisions with a Single Decision Maker

General Setting

• We have a set of possible *outcomes* X.

• There is a *utility function* $u : X \to \mathbb{R}$. We also think of u as a *payoff* function. The idea is that the utility function measures how we value the various outcomes. If $u(x) < u(y)$, then we prefer y to x, while if $u(x) = u(y)$, then we are indifferent between x and y.

• There is a set A of possible *actions* and a function $x : A \to X$. Action $a \in A$, leads to outcome $x(a)$. We define $v = u \circ x$. Thus action a outcome $x(a)$ and payoff $v(a) = u(x(a))$.

Rational Choice Assumptions I

• A decision maker fully understands A, X, x, u and v.

• A rational decision maker will (if possible) choose $a \in A^*$ to maximize v, i.e. a rational decision maker will choose $a^* \in A$ such that $v(a^*) \geq v(a)$ for all $a \in A$.[1]

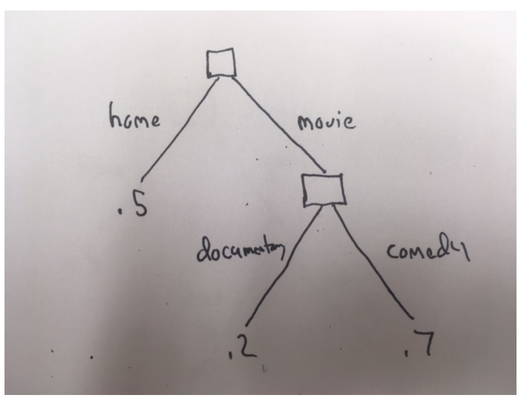
Two complications

• Compound decisions: Sometimes we will have to make a sequence of decisions, where later choices will depend on our earlier actions.

• Uncertainty: Sometimes the outcome will depend not only on our actions but also a random event.

Example 1 I first decide if I want to stay home or go to the movies. If he go to the movies I have to decide if I want to go to a comedy or a documentary. The payoff for staying home is .5, for going to a comedy is .7 and a documentary is .2.

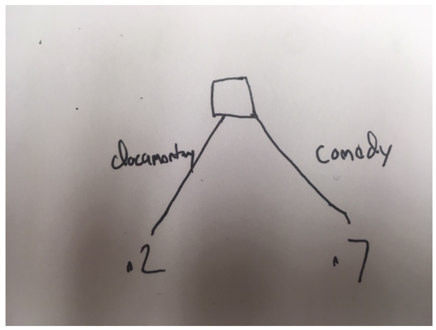
We form a *decision tree* to describe our choices.



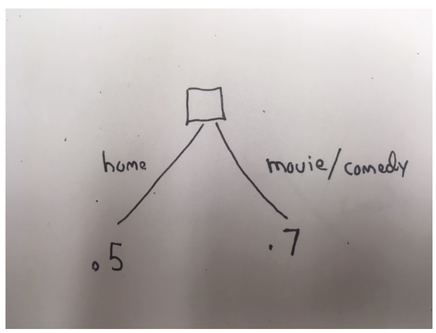
[1]This is not always possible, for example, if $A = (0, 1)$ and $v(a) = a$, then there is no maximum.

Square boxes indicate points where we need to make a decision. We make decisions at the top first and move down to later decisions. The numbers at the bottom indicate the payoff if we follow that path in the decision tree.

We can analyze our decisions using *backward induction*. The idea is that we start at the bottom with simple decisions and then work backward simplifying the earlier compound decisions. In this case we start with the decision of which movie I prefer.



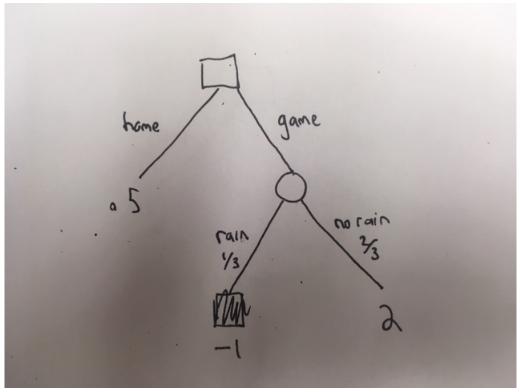
Clearly I prefer the comedy. Thus I can simplify my initial decision knowing that if I decide to stay home I will go to the comedy.



Now it is clear that I should choose to go to the movie. Thus decision is that I will go to the movie and choose the comedy.

Example 2 I need to decide if I will go to the Cubs game or stay home. If I stay home my payoff is .5. If go to game there is a 1/3 probability of rain. If it rains my payoff is -1. If it does not my payoff is 2.

We represent this decision as the following decision tree.



The circle represents a point where something is decided by chance. The main idea is that in this kind of situation I make my decision to maximize my expected utility. In this case my expected utility if I go to the game is

$$\frac{1}{3}(-1) + \frac{2}{3}(2) = 1.$$

. Thus, I am facing the choice of staying home with payoff .5 or going to the game and having an expected payoff of 1. I will choose to go to the game.

A *lottery* is a probability distribution over a set of outcomes. If the set of outcomes is finite listed as $x_1, \ldots, x_m$ a lottery is a set of real numbers $p_1, \ldots, p_m$ where $0 \leq p_i \leq 1$ for all i and $p_1 + \ldots + p_m = 1$.

Our choice in example 2 is to stay at home or choose the lottery with a 1/3 chance or rain or a 2/3 chance of no rain and the corresponding payouts.

Rational Choice Assumptions II (von Neumann–Morgenstern utilities)

• The utility of a lottery is it's expected value.

• A rational decision maker will choose an action that maximizes expected utility.

Theoretical Comments

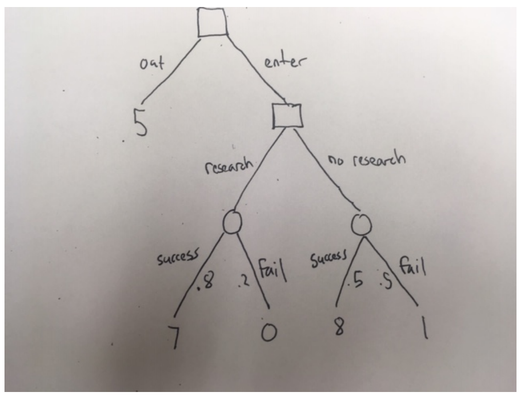
• In decision making (and game theory) where there is no random outcome, it is enough to think of our utility functions as being *ordinal*, i.e. while it matters if $u(x) > u(y)$ it doesn't matter if $u(x)$ is a little bigger that $u(y)$ or much bigger. All that matters it that we reflect that we prefer x to y. So we would make the same decision if we replaced u with $u_1(x) = e^{u(x)}$. We could not do this if we were taking expectations, then it may matter how big the difference is.

• There are times when this assumption may not be valid. One example is the *St. Petersburg Paradox*. You play a game with entry fee b. Suppose we flip a fair coin until heads occurs. Suppose heads on flip $t = N$. Then you will be paid 2^N cents (where this is your utility). The probability that $t = N$ is 2^{-N}. Thus the expected payout is

$$\sum_{N=1}^{\infty} 2^{-N}(2^N) = \sum_{N=1}^{\infty} 1 = +\infty.$$

In other words, whatever the entry fee b, you have an infinite expected payout. Does this mean a rational person would choose to play if the entry fee was, say, \$100,000? [2]

Example 3 A firm must decide to enter or stay out of a market. If they stay out they have a payoff of 5. If the enter they must decide whether or not to do market research. Doing research has a cost of 1. If they do research the probability of success is .8 and the probability of failure is .1. It they do not do research the probability of success is .5 and the probability of failure is .5. If they succeed the gross payoff (i.e. payoff before cost of research if applicable) is 8, while if they fail it is 1. We can represent this compound decision under uncertainty with the following decision tree.



We analyze this by backward induction. If we do research the expected payoff is 5.6, while if we do not do research the expected payoff is 4.5. Thus we would choose to do research. This simplifies our main decision. If we enter the market (and do research) we have an expected payoff of 5.6, while if we stay out we have an expected payoff of 5 so we will decide to enter the market and do research.

[2] One problem with this argument is the idea that at some point it's unreasonable to use money as a von Neumann–Morgenstern utility function. Would you be indifferent between being giving \$1,000,000 and flipping a coin where you get \$2,000,000 if it's heads and nothing if it's tails?