

#### Abstracts of contributed talks

- ▶ RICHARD B. ANGELL AND FRED MICHAEL, *The reconciliation of intuitionist and classical logic by T-operator*.  
Professor Emeritus, Department of Philosophy, Wayne State University, Detroit MI, 48202, USA.

*E-mail:* rbangell@bellatlantic.net.

Department of Philosophy, Brooklyn College, CUNY, Brooklyn, NY 11210, USA.

*E-mail:* fmichael@brooklyn.cuny.edu.

Heyting's Intuitionist Logic and classical two-valued logic differ on Double Negation and the Law of Excluded Middle. Classically not-not-P is logically equivalent to P. Heyting's Intuitionist Logic holds that P implies not-not-P, but not the converse. In classical logic every proposition is either true or false. In Heyting's logic some propositions about infinite sets are neither provably true nor provably false. Can these views co-exist in one logic?

A truth-operator 'T' is introduced for "It is true that ...". Trivalent truth-tables are established with '0' for "it is neither true nor false that". Disclaiming Tarski's convention T, P is not equivalent to TP. For if P is 0, TP is F. FP ("It is false that P") is defined as 'T ~ P'. This is not equivalent to '~ TP' for 'P is not true'. Classical connectives have the "strong sense" of Kleene's trivalent "strong tables". 'A tautology' is defined as any formula with T's but no F's in the final column of its truth-table. The following variants of classical Double Negation and Excluded Middle are all tautologies: ( $\sim\sim P \equiv P$ ), ( $T\sim\sim P \equiv TP$ ), ( $\sim T \sim TP \equiv TP$ ), and ( $\sim P \vee P$ ), ( $TP \vee \sim TP$ ), ( $\sim FP \vee FP$ ). Heyting's restrictions on DN and LEM hold by the fact that ( $FP \equiv TP$ ) and ( $TP \vee FP$ ) are both contingent. Both are F when P is 0. However, Non-Contradiction still holds. For  $\sim(TP \ \& \ FP)$  is a tautology, as is  $\sim(P \ \& \ \sim P)$  and its substitution instances  $\sim(TP \ \& \ \sim TP)$  and  $\sim(FP \ \& \ \sim FP)$ .

It is shown how these results are all provable from the axioms of Analytic Truth-logic, an extension with 'T' of A-logic [1].

[1] RICHARD BRADSHAW ANGELL, *A-LOGIC*, University Press of America, Lanham, MD, USA, 2002.

- ▶ KATALIN BIMBÓ, *A set theoretical semantics for full propositional linear logic*.  
Automated Reasoning Group, Institute of Advanced Studies, Australian National University,  
Bld. #115, Canberra, ACT 0200, Australia.

*E-mail:* bimbocsl.anu.edu.au.

We define a set theoretical semantics for full linear logic (i.e., classical linear logic including the exponentials). The *intensional connectives* ( $\circ$ ,  $\rightarrow$ ) and the *modal operator* ( $\diamond$ ) are modelled using operations (in a style similar to Urquhart's semantics for relevance logics, [3]). The *extensional connectives* ( $\wedge$ ,  $\vee$ ) are represented via a Galois connection that is defined almost exactly as in [1]. *Negation* ( $\sim$ ) is modelled using a binary relation  $\perp$ , however, we define the set theoretical operation and the canonical  $\perp$  relation differently than Dunn does in [2]. *Soundness* and *completeness* is proven. We also consider a few possible modifications of this semantics for linear logic, as well as for related substructural logics.

[1] K. BIMBÓ, *Semantics for structurally free logic LC+*, *Logic Journal of the IGPL*, vol. 9 (2001), pp. 557–571.

[2] J. M. DUNN, *Generalized ortho negation*, *Negation: a Notion in Focus*, (H. Wansing, editor), W. de Gruyter, New York, 1996, pp. 3–26.

[3] A. URQUHART, *Semantics for relevant logics*, *The Journal of Symbolic Logic*, vol. 37 (1972), pp. 159–169.

- ▶ JEFFREY BURDGES, *A signalizer functor theorem for groups of finite Morley rank*.  
Department of Mathematics, Rutgers University, Hill Center, Piscataway, New Jersey 08854, USA.

*E-mail:* burdges@math.rutgers.edu.

There is a longstanding conjecture, due to Gregory Cherlin and Boris Zilber, that all simple groups of finite Morley rank are simple algebraic groups. One of the major theorems in the area is Borovik's trichotomy theorem. The "trichotomy" here is a case division of the minimal counterexamples within odd type, i.e., groups whose Sylow 2-subgroup is divisible-by-finite. We introduce a characteristic zero notion of unipotence which can be used to obtain a connected nilpotent signalizer functor from any sufficiently non-trivial solvable signalizer functor. This result plugs seamlessly into Borovik's work to eliminate the assumption of tameness from his trichotomy theorem for odd type groups. This work also provides us with a form of Borovik's theorem for degenerate type groups.

- ▶ DENNIS F. CUDIA, *General inductive logic*.  
5343 Cunningham Road, Rockford, Illinois 61102, USA.

Denote the *gold standard effect for the cause*  $q$  by  $g(q)$  and define  $g(q)$  so that  $q$  is present if and only if the effect  $g(q)$  is present.  $g(q)$  is abbreviated by  $g$  and the absence of the effect  $g(q)$  is denoted by  $\sim g$ . Let  $H$  be a fixed finite non-empty set of instances each of the form  $(p, g)$  or  $(p, \sim g)$  or  $(\sim p, g)$  or  $(\sim p, \sim g)$  where  $p$  or  $\sim p$  denotes the presence or absence, respectively, of the effect  $p$ . Let  $n_{11}, n_{12}, n_{21}, n_{22}$ , respectively, be the number of instances of the form  $(p, g)$ ,  $(p, \sim g)$ ,  $(\sim p, g)$ ,  $(\sim p, \sim g)$ , respectively. Assume  $a$  and  $b$ , respectively, are positive rational numbers given by  $n_{11}/(n_{11} + n_{21})$  and  $n_{22}/(n_{12} + n_{22})$ , respectively. Denote the *likelihood table of*  $H$  by  $LT(H)$  and define  $LT(1, 1) = a$ ,  $LT(1, 2) = 1 - b$ ,  $LT(2, 1) = 1 - a$ ,  $LT(2, 2) = b$ . Denote a diagonal matrix by  $\text{diag}(u, v)$ , let  $C = \text{diag}(c, 1 - c)$ , let  $d = ac + (1 - b)(1 - c)$ , let  $D = \text{diag}(d, 1 - d)$  and denote the *predictive value table of*  $H$  by  $PV$  and define  $PV$  so that  $D \cdot PV = LT \cdot C$  where all inverses and multiplications are matrix operations. If  $a + b > 1$  then the graphs of  $PV(1, 1)$  and  $PV(2, 2)$ , respectively, are concave functions of  $c$  passing through  $(0, 0)$ ,  $(1, 1)$  and  $(0, 1)$ ,  $(1, 0)$ , respectively. Expressions are obtained for  $PV(1, 1) - c$  and  $PV(2, 2) - (1 - c)$ . An example from cardiology is discussed (see [1]) in which  $q$  is coronary heart disease and  $g$  is coronary arteriography and left ventriculography.

[1] R. G. MURRAY, ET AL., *Bayesian analysis of stress thallium-201 scintigraphy*, **European Journal of Nuclear Medicine**, vol. 6 (1981), pp. 201–204.

- ▶ NATASHA DOBRINEN, *A very weak distributive law and a related game in Boolean algebras*.  
Penn State University, 218 McAllister Building, University Park, PA 16802, USA.  
*E-mail:* dobrinen@math.psu.edu.

Infinitary distributive laws in Boolean algebras are of interest in their own right and also for their equivalences to some useful forcing properties. Jech pioneered the connections between distributive laws involving countable families of maximal incompatible subsets and related infinitary games between two players. Foreman, Kamburelis, and others have added to this body of knowledge.

We will review some results of Jech, Foreman, and Kamburelis along with some of our own regarding generalized distributive laws and related games. We will then present a generalized notion of weak distributivity due to Prikry, namely the hyper-weak  $(\kappa, \lambda)$ -distributive law, give a forcing equivalence, define a related game, and show that for certain pairs of cardinal numbers, the hyper-weak  $(\kappa, \lambda)$ -distributive law is equivalent to the non-existence of a winning strategy for the first player in the game. Under GCH, this equivalence holds for all pairs  $\kappa \geq \lambda$ . It is consistent with ZFC that, for all  $\kappa \geq \lambda$ , this game is undetermined, via  $\kappa^+$ -Suslin trees.

- ▶ ALFIO GIARLOTTA, *The representability number of a chain*.  
Department of Mathematics, University of Illinois, Urbana IL 61801, USA.  
*E-mail:* giarlott@math.uiuc.edu.  
Department of Economics and Quantitative Methods, University of Catania, Catania 95129, Italy.

For any pair of linear orderings  $(L, M)$ , define the  $M$ -representability number of  $L$ , denoted by  $\text{repr}_M(L)$ , as the least ordinal  $\alpha$  such that  $L$  can be embedded into the lexicographic power  $M_{lex}^\alpha$ . We compute  $\text{repr}_M(L)$  for some pairs of linear orderings  $(L, M)$ , paying particular attention to the case  $M = \mathbb{R}$ .

A chain  $L$  is  $M$ -large if  $\text{repr}_2(L) = \text{repr}_M(L)$ ; a pair of chains  $(L, M)$  is *lexicographically incomparable* if  $L$  is  $M$ -large and  $M$  is  $L$ -large. We give some examples of pairs of chains which are lexicographically incomparable.

- ▶ LEV YU. GLEBSKY AND YEVGENIY GORDON, *On a class of word sets and extension of monadic second order language*.  
IICO-UASLP, Av.Karakorum 1470, Lomas 4TA Session, San Luis Potosi SLP 7820 Mexico.  
*E-mail:* glebsky@cactus.iico.uaslp.mx.  
Eastern Illinois University, Department of Mathematics, 600 Lincoln Ave., Charleston, IL, 61920-3099, USA.  
*E-mail:* cfyig@eiu.edu.

A new class  $\Pi_L$  of languages (word sets) with polynomial algorithms for the membership problem will be discussed in the talk. This class is a generalization of the class  $\Pi$  of languages introduced by Yu. V. Glebsky in “A certain class of word sets”, ((Russian) *Izvestia Vyssh. Uchebnyh. Zavediny. Radiofizika* 13 (1970), 1256–1258) and thus includes all regular languages. But  $\Pi_L$  includes also some languages, such as Correct Bracket Structures and the set of palindromes, that are not in  $\Pi$ . We consider also a similar to  $\Pi$  class of languages for words on trees. We introduce also a weak second order language  $\text{MSOC}_L$  that generalizes the well-known J. R. Büchi’s and M. O. Rabin languages for finite automata and finite automata on trees. Since the class  $\Pi_L$  is closed only about Boolean operations but not about word projection, there does not exist a one-to-one correspondence between the  $\Pi_L$  languages and formulas of  $\text{MSOC}_L$ . But we describe a subset  $K_L$  of  $\text{MSOC}_L$  that is in one-to-one correspondence with  $\Pi_L$  languages. For any formula  $\varphi \in K_L$  there exists an algorithm, checking if  $M \models \varphi$  in polynomial time with respect to size  $M$  in the standard interpretation. This implies the following statement: a problem about graphs which can be formulated in  $K_L$  has polynomial time algorithm for graphs with bounded tree depth. For example, maximum planar subgraph problem can be formulated on  $K_L$ .

- ▶ JEFFRY L. HIRST, *Milliken’s Theorem, ultrafilters, and reverse mathematics (Preliminary Report)*.  
Department of Mathematical Sciences, Appalachian State University, Boone, NC 28608, USA.  
*E-mail:* jlh@math.appstate.edu.

Milliken’s Theorem [2] is a theorem of infinite combinatorics combining aspects of Ramsey’s Theorem and Hindman’s Theorem [1]. This talk will explore the connections between Milliken’s theorem and the existence of certain ultrafilters on countable Boolean algebras.

[1] NEIL HINDMAN, *Finite sums from sequences within cells of a partition of  $N$* , *Journal of Combinatorial Theory Series A*, vol. 17 (1974), pp. 1–11.

[2] KEITH R. MILLIKEN, *Ramsey’s theorem with sums or unions*, *Journal of Combinatorial Theory Series A*, vol. 18 (1975), pp. 276–290.

- ▶ TETSUYA ISHIU, *Club guessing sequences and their applications*.

Department of Mathematics, 103 Multipurpose Science and Technology Building, University of California, Irvine, Irvine, CA 92697-3875, USA.

*E-mail:* tishiu@math.uci.edu.

A club guessing sequence was defined by Shelah. Its definition is similar to the ones of  $\diamond$  and  $\clubsuit$ . But unlike these principles, there always exists a club guessing sequence on every regular cardinal above  $\aleph_1$ . It was very effectively used in various arguments. For example, it played an important role in pcf theory. It also has an application to general topology, which was found by a joint work with Fernando Hernández-Hernández. I will talk about my recent research on this interesting mathematical object.

- ▶ JUI-LIN LEE, *On Gödel's completeness theorem*.

National Chung-Cheng University, Philosophy Department, Chia-Yi, Taiwan 621.

*E-mail:* jlllee@phil.ccu.edu.tw.

In the first part “Skolemizing Gödel’s completeness theorem” I give a simple proof of completeness theorem. It suffices to consider the prenex normal form case. Then the simple cases  $\Pi_1$  and  $\Sigma_1$  can be easily proved: use canonical model and Lindenbaum’s trick on diagram for  $\Pi_1$  case (the universal quantifier then may stand for “for all (equivalence classes of) variable free terms”) and introduce new constant symbols for  $\Sigma_1$  case (the key in Henkin’s proof). Completeness theorem then seems trivial for one may introduce Skolem functions to convert any  $\forall x \exists y \varphi(x, y)$  into  $\forall x \varphi(x, f(x))$  and then it can always be reduced to the  $\Pi_1$  case. It is not so simple. To preserve the consistency for introducing Skolem functions one needs a careful proof-theoretic analysis to prove Herbrand’s theorem and then the “Theorem on Functional Extensions” [3] (pp. 55–56). I will avoid proving “Theorem on Functional Extensions” and modify Henkin’s method instead: The proof of completeness theorem can be done by using Skolem functions to index new constant symbols and introducing new constant symbols in a different type of well-ordering.

In the second part “Can a finitist understand the proof of completeness theorem for monadic logic?” I will discuss the proof of completeness theorem for monadic logic without AC (or Maximal Ideal Theorem).

[1] W. HODGES, *Model theory*, Cambridge University Press, 1993.

[2] S. SHAPIRO, *Classical logic II: higher-order logic*, *The Blackwell Guide to Philosophical Logic*, L. Goble (editor), Blackwell Publishers Ltd., 2001, pp. 33–54.

[3] J. R. SHOENFIELD, *Mathematical Logic*, Addison-Wesley Press, 1967.

- ▶ DANIEL LEIVANT, *The constructive arithmetical hierarchy*.

Computer Science Department, Indiana University, Bloomington, IN 47405, USA.

*E-mail:* leivant@cs.indiana.edu.

Classically, the same algorithms are provably-convergent using  $\Sigma_k^0$ -induction as  $\Pi_k^0$ -induction, yielding a neat correspondence between function provability using induction for  $k$  quantifier-alternations, and definability by rank- $k$  primitive-recursion. This simple picture collapses when the logic is constructive; in particular,  $\Pi_1^0$ -induction is much weaker than  $\Sigma_1^0$ -induction, and induction for arbitrary prenex formulas is no stronger than  $\Sigma_2^0$ -induction [3].

We account for these phenomena, and related ones, by developing a constructive analog to the arithmetical hierarchy, focusing on the computational power of restricted induction, rather than on descriptive power (as in [1]). Using the intrinsic-reasoning framework of [2], with  $N$  as a primitive predicate (as did Peano in 1889!), we define a formula-as-type homomorphism that maps formulas to simple types over  $N$  (no dependent types), and we calibrate formulas by the implication rank of that type.

The functions provable constructively (that is, over minimal or intuitionistic logic) using rank- $k$  induction are seen to be precisely the functions defined by rank- $k$  primitive recursion.

Since formulas of PA are interpreted by relativizing quantifiers to  $N$ , the formulas in  $\Pi_1^0$ ,  $\Sigma_1^0$ ,  $\Sigma_2^0$ ,  $\Pi_{k+2}^0$  and  $\Sigma_{k+3}^0$  correspond indeed to ranks 0, 1, 1, 2 and 2, respectively. However, if double negations are introduced to capture the full classical classes, then the rank of formulas grows with quantifier alternation.

Research partially supported by NSF grant CCR-9309824.

[1] WOLFGANG BURR, *The intuitionistic arithmetical hierarchy*, Preliminary manuscript, 2000.

[2] DANIEL LEIVANT, *Intrinsic reasoning about functional programs I: first order theories*, *Annals of Pure and Applied Logic*, vol. 114 (2002), pp. 117–153.

[3] KAI F. WEHMEIER, *Fragments of HA based on  $\Sigma_1$ -induction*, *Archive for Mathematical Logic*, vol. 37 (1997), pp. 37–49.

- ▶ YEVGENIY MAKAROV, *Another characterization of provably recursive functions*. Department of Computer Science, Indiana University, Bloomington, IN 47405-7104, USA. E-mail: emakarov@cs.indiana.edu.

A new characterization of provably recursive functions of first-order arithmetic is described. Its main feature is using only terms consisting of 0, the successor  $S$  and variables in the quantifier rules, namely, universal elimination and existential introduction.

Consider Peano arithmetic with functional symbols for all primitive recursive functions, and defining equations for them. Consider also a set  $F$  of new functional symbols and a set  $P$  of equations for them. These equations don't have to be primitive recursive, and they even don't have to define total functions. For  $f \in F$ , the function defined by  $P$  for  $f$  is

$$\{(n, m) \mid f(n) = m \text{ is provable from } P \text{ in equational logic}\}.$$

If  $P$  does not prove  $m = n$  for closed terms  $m, n$  consisting of 0,  $S$  only then this is indeed a function.

We claim that a function defined by  $P$  for  $f$  is provably recursive iff the formula  $\forall x \exists y f(x) = y$  is provable from  $P$  in Peano arithmetic using only 0,  $S$  and variables in the rules of universal elimination and existential introduction. (Natural deduction is used; we also have the the following rules for equality: reflexivity and  $A[t]$  with  $t = s$  imply  $A[s]$  for any formula  $A$  and terms  $t, s$ .) There is no limitation on induction.

The proof is very similar to the one of the analogous statement about Intrinsic Theories described in [1]. In fact, one direction uses Intrinsic Theories directly.

[1] DANIEL LEIVANT, *Intrinsic reasoning about functional programs I: first order theories*, *Annals of Pure and Applied Logic*, vol. 114 (2002), no. 1–3, pp. 117–153.

- ▶ KERRY OJAKIAN, *Ramsey lower bounds in bounded arithmetic*. Carnegie Mellon University, Department of Mathematics, 6113 Wean Hall, Pittsburgh, PA 15217, USA.

E-mail: keo@andrew.cmu.edu.

Working in bounded arithmetic, Pudlak [2] proves a Ramsey upper bound statement. We investigate lower bounds, both non-constructive (via the probabilistic method) and constructive ones. The weak pigeon hole principle (WPHP) is key in much of this work; it states that there is no injection from  $[2n]$  to  $[n]$ . For the multi-color Ramsey statement (call it “Ram”), formalizing the constructive lower bound yields a sort of “reversal” to Pudlak’s result. He shows that  $WPHP \rightarrow Ram$ . We show that  $Ram \rightarrow WPHP$ , using an idea from Krajicek [1].

[1] JAN KRAJICEK, *On the weak pigeonhole principle*, *Fundamenta Mathematicae*, vol. 170 no. 1–3, (2001), pp. 123–140.

[2] PAVEL PUDLAK, *Ramsey's theorem in bounded arithmetic*, **4th Workshop, computer science logic**, Lecture notes in computer science, vol. 533, Springer-Verlag, Berlin Heidelberg, 1991, pp. 308–317.

- PAULO OLIVA, *On the extraction of polynomial-time algorithms from ineffective proofs in feasible analysis*.

Department of Computer Science, University of Aarhus, Ny Munkegade, DK-8000 Aarhus C, Denmark.

*E-mail*: pbo@brics.dk.

Consider as a *base for feasible analysis* Cook and Urquhart's [1] system  $CPV^\omega$  extended with the axiom of arithmetical choice for quantifier-free formulas QF-AC. This system has a simple functional interpretation (via negative translation) in  $IPV^\omega$ , which allows one to extract polynomial-time computable realizers from proofs of  $\Pi_2^0$ -theorems (i.e., formulas of the kind  $\forall x \exists y A(x, y)$ ,  $A$  being quantifier-free) in  $CPV^\omega + QF-AC$ . In [3] an extension of this functional interpretation to include a 'feasible' formulation of weak König's lemma for  $\Pi_1^0$ -definable trees is given by extending  $IPV^\omega$  with a simple form of binary bar recursion  $\mathcal{B}$ , i.e., it is shown that the theory  $CPV^\omega + QF-AC + \Pi_1^0-WKL^\omega$  has a functional interpretation (via negative translation) in  $IPV^\omega + \mathcal{B}$ . Moreover, in [3] it is also proven that the type one terms of the system  $IPV^\omega + \mathcal{B}$  are polynomial-time computable. This gives a procedure for extracting polynomial-time realizers from proofs of  $\Pi_2^0$ -theorems in  $CPV^\omega + QF-AC + \Pi_1^0-WKL^\omega$ . The  $\Pi_2^0$ -theorems are even allowed to make use of 0-1 oracles as free-variables.

The base system  $CPV^\omega + QF-AC$ , which when extended with  $\Pi_1^0-WKL^\omega$  is strong enough for proving e.g., the Heine/Borel covering lemma, is an extension to higher types of a subsystem of Ferreira's [2] BTFA (Base Theory for Feasible Analysis), for which  $\Pi_2^0$ -conservation of WKL has been non-constructively proven.

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[1] S. COOK AND A. URQUHART, *Functional interpretations of feasibly constructive arithmetic*, *Annals of Pure and Applied Logic*, vol. 63 (1993), pp. 103–200.

[2] F. FERREIRA, *A feasible theory for analysis*, *The Journal of Symbolic Logic*, vol. 59 (1994), no. 3, pp. 1001–1011.

[3] P. OLIVA, *Polynomial-time algorithms from ineffective proofs*, Submitted, 2003.

- MAURICIO OSORIO AND JUAN ANTONIO NAVARRO, *Modal Logic  $S5_2$  and  $FOUR$* , Universidad de Las Americas, Puebla, Santa Catarina Martir s/N, Departamento de Ingeniería en Sistemas, Cholula Puebla 72820 Mexico.

*E-mail*: josorio@mail.udlap.mx.

Departamento de Matematicas, Santa Catarina Martir S/N, San Andres Cholula, Puebla 72820 Mexico.

*E-mail*: ma108907@mail.udlap.mx.

Our connectives for  $S5$  are  $\mathbf{b}$  (Box),  $\sim$  (negation) and  $\vee$  (or).  $A \rightarrow B$ ,  $A \ \& \ B$ , and  $d A$  are considered as abbreviations in the standard way.

Consider the axiom schemata:

$$F1 := d A \rightarrow b A,$$

$$F2 := ((d A1 \ \& \ d A2) \ \& \ b \sim (A1 \ \& \ A2)) \rightarrow b (A1 \vee A2), \quad \text{and so on.}$$

For every Natural number  $i$  the reader can picture the axiom schema  $F_i$ .

For every Natural number  $i$ ,  $S5_i$  denotes  $S5$  extended with the schema  $F_i$ .

We show that  $S5_i$  is determined by the class of reflexive, transitive, and symmetric frames of size at most  $i$ .  $S5_1$  is the "not very interesting" (sometimes) called Trivial modal logic. This family of logics define a strictly descending chain, where  $S5_i$  is strictly stronger than

$S5_{(i+1)}$ . Moreover, we present a translation of  $S5_2$  into the well known bilattice FOUR that preserves theorems. FOUR has the for values  $\{0, 1, t, f\}$ . We write  $\& 1$  and  $\vee 1$  to denote the meet and join operators w.r.t. truth ordering. We have also two negation operators  $-1$  (negation) and  $-2$  (conflation):

$$\begin{aligned} -1(t) &:= f, & -1(f) &:= t, & -1(1) &:= 1, & \text{and} & & -1(0) &:= 0. \\ -2(t) &:= t, & -2(f) &:= f, & -2(1) &:= 0, & \text{and} & & -2(0) &:= 1. \end{aligned}$$

We define the function  $f$  from modal formulas into FOUR formulas as follows:

$$\begin{aligned} f(A) &:= A \text{ if } A \text{ is atomic.} \\ f(b A) &:= f(A) \ \& 1 \ - f(A). \\ f(\sim A) &:= -1 \ -2 f(A). \\ f(A \vee B) &:= f(A) \vee 1 f(B). \end{aligned}$$

A  $t$ -tautology is a formula that evaluates always to  $t$ . We show that  $A$  is theorem of  $S5_2$  iff  $f(A)$  is a  $t$ -tautology.

We observe that  $S5_2$  agrees with  $S5$  in a non trivial fragment of formulas, interestingly enough for applications in computer science. Hence, we show that a small but relatively interesting fragment of  $S5$  can be translated to FOUR.

- ▶ WIM RUITENBURG, *Intuitionistic theories whose Kripke models are all T-normal ones.* Department of Mathematics, Marquette University, P.O. Box 1881, Milwaukee WI 53201, USA.

*E-mail:* wimr@mscs.mu.edu.

Given a set of sentences  $T$ , a Kripke model is  $T$ -normal, or locally  $T$ , if all its node structures are (classical) models of  $T$ . Samuel Buss constructs a theory  $H(T)$  which is sound and complete for the locally  $T$  Kripke models. We show that if, additionally, all models of  $H(T)$  are locally  $T$ , then  $H(T)$  is axiomatizable by geometric sentences. This answers a question implied by Wolfgang Burr's Mathematical Review MR 2001h:03115.

[1] SAMUEL R. BUSS, *Intuitionistic validity in T-normal Kripke structures*, *Annals of Pure and Applied Logic*, vol. 59 (1993), pp. 159–173.

- ▶ KSENIJA SIMIC, *Reverse mathematics of Hilbert spaces.* 221 Gross St., Pittsburgh, PA 15224, USA.

*E-mail:* ksimic@andrew.cmu.edu.

To date, little research has been done in the area of Hilbert spaces from the point of view of weak subsystems of second order arithmetic. In this talk, I will give the definitions and introduce basic concepts relevant to developing the theory of Hilbert spaces in this framework, and then state some results regarding orthogonal projections and closed sets. I will also discuss the mean ergodic theorem, and show that it is equivalent to arithmetic comprehension over the base theory  $RCA_0$ .

- ▶ STEPHEN M. WALK, *Blanketing functions and a conjecture about array computable degrees.* Department of Mathematics, ECC 139, 720 4th Avenue South, St. Cloud State University, St. Cloud, Minnesota 56301-4498, USA.

*E-mail:* smwalk@stcloudstate.edu.

We define a *blanketing function* for a c.e. degree  $\mathbf{a}$  to be a function  $f \leq_{wt} K$  that dominates all  $\mathbf{a}$ -computable functions. The concept comes from Downey, Jockusch, and Stob's definition of *array computable* [1]: A degree  $\mathbf{a}$  is array computable, by definition, if and only if such a function  $f$  exists.

Every blanketing function must have high c.e. degree, and it is natural to wonder *which*

high c.e. degrees contain blanketing functions. It is shown in [1] that  $\theta'$  contains a blanketing function for every a.c. degree, and similarly every high c.e. degree contains a blanketing function for  $\theta$ . But it is not immediately clear whether any *incomplete* c.e. degree contains a blanketing function for any *noncomputable* a.c. degree.

We show that every high degree does contain a blanketing function for some noncomputable a.c. degree; that, similarly, every a.c. degree is “blanketed” by some incomplete degree; and that no two a.c. degrees are “blanketed” by the same set of degrees, even when we restrict our attention to high degrees above *both* of them. We present negative results in the same vein and consider larger questions related to a result of Kummer [2] and to a conjectured duality between the high degrees and the a.c. degrees. (All degrees considered in these results are c.e.)

[1] RODNEY G. DOWNEY, CARL G. JOCKUSCH, and MICHAEL STOB *Array nonrecursive degrees and genericity*, *Computability, enumerability, unsolvability: Directions in recursion theory* (S. B. Cooper, T. A. Slaman, and S. S. Wainer, editors), London Mathematical Society Lecture Series, no. 224, Cambridge University Press, 1996, pp. 93–104.

[2] MARTIN KUMMER *Kolmogorov complexity and instance complexity of recursively enumerable sets*, *Siam Journal on Computing*, vol. 25 (1996), pp. 1123–1143.

- FRANK WROBLEWSKI, *The completion of classical propositional-quantificational logic accommodates indeterminacy*.  
1662 Coughlin St., Laramie, WY 82072, USA.  
*E-mail*: fwroblewski@hotmail.com.

Since Aristotle pondered a third truth-value for indeterminate, i.e., neither true nor false propositions, the true-false dichotomy has been disputed. Classical logic is tasked to guide truth preserving discourse and performs well with determinate, i.e., true or false propositions, but founders on truth-value gaps occasioned by indeterminacies such as Heisenberg’s uncertainty and ‘ $x \in y$ ’ mathematics. Charles S. Peirce, a firm believer in unavoidable indeterminacy, hoped to complete classical logic by extending truth tables to three truth-values. But truth tables are tabulations derived from interrelated received hypotheses where tinkering violates classical logic.

We complete classical logic via Sheffer’s reduction. Two hypotheses are found wanting: the law of bivalence doesn’t exhaust the truth-values, and Sheffer’s hypothesis falsifying joint-denial neglects the false-conferring condition where one proposition is the denial of the other. Accordingly, the truth-values are exhausted via the law of excluded middle, yielding the complementary denial of ‘true or false,’ namely, the third truth-value ‘neither true nor false.’ By completing Sheffer’s deficient hypothesis, propositional logic is completed wherein the core two-valued logic is preserved to become the special case for determinate propositions. It remains to deduce the truth conditions conferring the ‘indeterminate’ truth-value on joint-denial, and deduce the truth conditions conferring each of the three truth-values on the other connectives.

The quantification hypotheses cleansed of Quine’s expedient closure, with truth predicates restored, suffice to deduce quantificational truths accommodating indeterminacy. Thus the correct form of existential instantiation is deduced. Quantification proves idle for determinate propositions. Undecidable propositions bear two or three alternate truth-values; e.g., the identity ‘ $x = y$ ’ is true or indeterminate.

We assume familiarity with Quine’s *Mathematical Logic*, especially Sheffer’s reduction.

- GUOHUA WU, *Complementing the cappable degrees in the difference hierarchy*.  
School of Mathematical and Computing Sciences, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand.  
*E-mail*: wu@mcs.vuw.ac.nz.

In [1], Downey proved that the diamond lattice can be embedded into the d.c.e. degrees preserving 0 and 1. Wu proves in [2] that such diamond embeddings can also be obtained by showing that there are c.e. degrees  $\mathbf{a}$ ,  $\mathbf{c}$  and a d.c.e. degree  $\mathbf{d}$  such that  $\mathbf{c}$  cups  $\mathbf{d}$  to  $\mathbf{0}'$ , caps  $\mathbf{a}$  to  $\mathbf{0}$ , and  $\mathbf{d}$  is isolated by  $\mathbf{a}$ . In this talk we will prove that any nonzero cappable degree has an isolated degree as its complement. As a consequence, a c.e. degree is cappable if and only if it has a nonzero degree as its complement. This gives a new characterization of cappable degrees. (Recently, Wu [3] showed that any nonzero cappable degree can also have a nonisolated degree as its complement.) This is a joint work with Rod Downey (Victoria University of Wellington) and Angsheng Li (Chinese Academy of Sciences).

[1] R. G. DOWNEY, *D.r.e. degrees and the nondiamond theorem*, *Bulletin of the London Mathematical Society*, vol. 21 (1989), pp. 43–50.

[2] G. WU, *Isolation and diamond embeddings*, *The Journal of Symbolic Logic*, vol. 67 (2002), pp. 1055–1064.

[3] ———, *Diamond embeddings into the d.c.e. degrees with 0 and 1 preserved*, *Proceedings of the 7th and 8th Asian Logic Conferences*, World Scientific, accepted, 2003.