Abstracts of invited plenary talks

▸ GREGORY CHERLIN, *Simple groups of finite Morley rank: The Borovik Program*.
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Groups interpretable in uncountably categorical structures inherit a notion of dimension known as Morley rank, and the rank is finite in this context. From a group theoretical point of view it is natural to ask for an explicit classification of the simple (nonabelian) ones. All known simple groups of finite Morley rank are definable over algebraically closed fields (in short, are "algebraic"), and it has been conjectured that there are no others. As the classification of the simple algebraic groups is known explicitly (generally expressed in terms of "Dynkin diagrams"), the conjecture has very explicit content. Borovik suggested that some aspects of the problem might be analysed using methods which were used successfully in the classification of the finite simple groups. This program has been pursued with some substantial success. I intend to present the Borovik program, and describe some of its successes and some of its limitations. While the work has been going on for more than a decade, some recent work involving Altinel, Berkman, Borovik, Frecon, Jaligot, Nesin, Poizat, and Wagner has broadened and simplified the picture. (Some of this work is in progress, and its final status may not be clear at the time of this report.)

In this work methods of finite group theory are combined with ideas coming from the theory of algebraic groups, with occasional injections of pure model theory, the latter most powerfully represented in recent times by work of Wagner.

▸ ALAN DOW, *MAD families and topological spaces*.
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Is there a compact sequential space which is not Frechet? Yes, because the natural psi-space extension of the integers N associated with an almost disjoint (mad) family is locally compact and its one-point compactification is sequential but not Frechet. Are there two compact Frechet spaces whose product is not Frechet? Yes [P. Simon], construct a special mad family which can be written as the union of two nowhere maximal almost disjoint families. We will investigate examples of questions about separable compact spaces that seem closely related to mad families and raise questions about mad families that result.

▸ YURI GUREVICH, *Kolmogorov machines and related issues*.
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Turing arrived at his computation model by analyzing computations performed by human computers. Kolmogorov arrived to his computation model by analyzing computation as a physical process. As far as computable functions are concerned, the Kolmogorov model is equivalent to the Turing model. But as far as computational complexity is concerned, the more flexible Kolmogorov model offered substantial advantages. Levin utilized the advantages to construct an algorithm for any given NP problem that is optimal up to a constant factor. One important question is how well the Kolmogorov machine model reflects the intuitive notion of algorithm.

▸ LEO HARRINGTON, *Anaximander's Saying*.
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This talk will present a reading of fragment B1 of the PreSocratic philosopher Anaximander. This fragment has been called "the incipient saying of being". The reading will be entirely mathematical. This is not intended to suggest that the saying itself is mathematical, but only to suggest that such a mathematical reading might help one better hear what the saying may be trying to say.

The reading is based on a key word 'apeiron' from another Anaximander fragment (A9) plus the PreSocratic philosopher Parmenides' description of 'peiras' (in fragment B8.42–44) as 'like the bulk of a well-rounded sphere everywhere from the center equally matched' which this reading will take (in the context of group actions) as 'like an orbit under a stabilizer subaction'.

In mathematical logic, group actions provide an abstract way of looking at what model theory deals with more concretely; group actions (or rather stabilizer subactions) have the advantage, for the purposes of this reading, of not being incipiently tied to a definite interpretation of a definite language. A main feature of the reading proposed in this talk is a rather familiar maneuver from model theory and group actions (and elsewhere) involving the reversal of a thing (in orbit) with its base (a stabilizer), allowing the identification as 'the same' of apparently disparate types of things, while leaving the dependency of 'not the same' on the definiteness of the choice of interpretation of the choice of language.

▸ ULRICH KOHLENBACH, *Proof theoretic applications to functional analysis*.
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We give a survey of recent progress in the area of proof mining. 'Proof mining' is concerned with the extraction of new information from proofs by means of proof theoretic techniques (so-called proof interpretations) such as Gödel's functional interpretation and specially designed variants thereof. This approach has recently had substantial applications in L1-approximation [4] and metric fixed point theory [1, 2, 3]. The applications in fixed point theory systematically yielded not only new effective bounds but even new qualitative results on the uniformity of asymptotic regularity of nonexpansive functions which only partly had been known before. In studying the logical structure of these applications we recently succeeded in proving rather general meta-theorems which guarantee the existence (as well as the extractability of effective versions) of such uniformity results under very broad and easy to check logical conditions. Our meta-theorems deal with general classes of spaces like metric spaces, hyperbolic spaces, normed linear spaces and uniformly convex spaces and classes of functions between such spaces like nonexpansive or directionally nonexpansive functions.

[1] U. Kohlenbach, *A quantitative version of a theorem due to Borwein-Reich-Shafrir*, ***Numerical Functional Analysis and Optimization***, vol. 22 (2001) pp. 641–656.

[2] U. Kohlenbach, *Uniform asymptotic regularity for Mann iterates*, to appear in ***Journal of Mathematical Analysis and Applications***.

[3] U. Kohlenbach, L. Leuştean, *Mann iterates of directionally nonexpansive mappings in hyperbolic spaces*, to appear in ***Abstract and Applied Analysis***.

[4] U. Kohlenbach, P. Oliva, *Effective bounds on strong unicity in L_1-approximation*, ***Annals of Pure and Applied Logic***, vol. 121 (2003), pp. 1–38.

[5] U. Kohlenbach, P. Oliva, *Proof mining: a systematic way of analysing proofs in analysis*, to appear in ***Proceedings of the Steklov Institue of Mathematics***.

▸ PHOKION G. KOLAITIS, *Constraint satisfaction, finite-variable logics, and treewidth*.
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Constraint satisfaction problems constitute a broad class of algorithmic problems that are ubiquitous in several different areas of computer science. In their full generality, constraint satisfaction problems are NP-complete and, thus, presumed to be algorithmically intractable. For this reason, extensive research efforts have been devoted to the pursuit of "islands of tractability" of constraint satisfaction, that is, special cases of constraint satisfaction problems for which polynomial-time algorithms exist.

The aim of this talk is to present an overview of recent advances in the investigation of the computational complexity of constraint satisfaction with emphasis on the connections between "islands of tractability" of constraint satisfaction, definability in finite-variable logics, and structures of bounded treewidth.

▸ LEONID A. LEVIN, *Forbidden information*.
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Gödel Incompleteness Theorem falls short of implications usually attributed to it. Closing this loophole involves quite technical use of Kolmogorov Complexity (unrelated to, well studied before, complexity quantifications of the usual Gödel effects). Similar problems and answers apply to other unsolvability results for tasks with non-unique solutions, such as, e.g., non-recursive tilings.

D. Hilbert asked if the formal arithmetic can be consistently extended to a complete theory. The question was somewhat vague since an obvious answer was 'yes': just add to the axioms of Peano Arithmetic (PA) a maximal consistent set, clearly existing albeit hard to find. K. Gödel formalized this question as existence among such extensions of recursively enumerable ones and gave it a negative answer (apparently, never accepted by Hilbert). Its mathematical essence is the lack of total recursive extensions of universal partial recursive predicate.

As is well known, the absence of algorithmic solutions is no obstacle when the requirements do not make a solution unique. A notable example is generating strings of linear Kolmogorov complexity, e.g., those that cannot be compressed to half their length. Algorithms fail, but a set of dice does a perfect job!

Thus, while r.e. sets of axioms cannot complete PA, the possibility of completion by other simple means remained open. In fact, it is easy to construct an r.e. theory R that, like PA, allows no consistent completion with r.e. axiom sets. Yet, it allows a recursive set of PAIRS of axioms such that random choice of one in each pair assures such completion with probability .99.

The reference to randomized algorithms seems rather special. However, the impossibility of a task can be formulated more generically. In 1965 Kolmogorov defined a concept of Mutual Information in two finite strings. It can be refined and extended to infinite sequences, so that it satisfies conservation laws: cannot be increased by deterministic algorithms or in random processes or with any combinations of both. In fact, it seems reasonable to assume that no physically realizable process can increase information about a specific sequence.

In this framework one can ask if the Hilbert-Gödel task of a consistent completion of a formal system is really possible for PA, as it is for an artificial system R just mentioned. A negative answer follows from the existence of a specific sequence that has infinite mutual information with ALL total extensions of a universal partial recursive predicate. It plays a role of a password: no substantial information about it can be guessed, no matter what methods are allowed. This "robust" version of Incompleteness Theorem is, however, trickier to prove than the old one.

▶ MICHAEL MAKKAI, *A new set theory*.
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In my talk "Towards a Categorical Foundation of Mathematics" at the Logic Colloquium '95 (see the paper in Springer Lecture Notes in Logic 11, 1998, and the references therein), I gave a statement, which remained incomplete in several ways, of the "Structuralist Foundation of Abstract Mathematics" (SFAM). The main part of the talk is a report on the progress made in the meantime in the program of the SFAM; a general introduction to the main starting ideas is also to be given.

It is the "underlying" language of the SFAM, called First Order Logic with Dependent Sorts (FOLDS), without the specific axioms, that was described fully in *loc. cit.* and in accompanying references; on the universe, only a start was made. As a result of successive developments, starting with suggestions made by John Baez and James Dolan, the definition (in ordinary set-theoretic terms) of (the main part of) the universe is given in the concept named in the title of paper "The multitopic omega-category of all [small] multitopic omega-categories" (September, 1999; at: www.math.mcgill.ca/makkai/).

SFAM is closely related to (although *not* dependent on) the subject of "Higher Dimensional Categories" (HDC's), pursued by many researchers interested in applications in topology, physics, quantum groups, etc. There are various (partial) definitions of "HDC", some, but not all, of which are possible (partial) alternatives for the universe of SFAM. The subject of comparing alternative (partial) definitions of the universe of SFAM is an important one; I approach it by a use of the framework of FOLDS.

▶ RALPH MCKENZIE, *The interplay between structural insight and algorithmic problems in general algebra*.
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In 1992, B. Hart, S. Starchenko and M. Valeriote proved Vaught's conjecture for varieties. Every variety (equationally defined class of algebras) of countable signature has either a continuum, or at most countably many, non-isomorphic denumerable models. They proved that every such variety with few models admits a decomposition into a product of a combinatorial variety and an affine variety (equivalent to a variety of all modules over some ring).

In 1986, R. McKenzie and M. Valeriote proved that every locally finite variety with decidable first order theory admits a decomposition into a product of a combinatorial variety, an affine variety, and a discriminator variety. The proof afforded an algorithm which inputs a finite algebra **F**, and outputs a finite ring **R** with unit, so that the variety $\mathcal{W}$ generated by **F** is decidable iff the theory of unital modules over **R** is decidable.

In 1993, R. McKenzie proved that there is no algorithm to determine, for a finite algebra **F**, whether the variety $\mathcal{W}$ generated by **F** is residually finite. Likewise there is no algorithm to determine if $\mathcal{W}$ is finitely axiomatizable.

All these results are related by dependence on the techniques (or at least the insights) flowing from the structure theory for finite algebraic systems called "tame congruence theory". In this talk, I will attempt to convey the substance of this theory, and its contribution to these results.

▶ DON PIGOZZI, *Abstract algebraic logic and the algebraic hierarchy*.
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All logics have both an *assertional* and an *equivalential* aspect that are represented respectively by the relations of consequence and logical equivalence. The connection between them is very close in the case of the familiar logics: each is simulated by the other in a straightforward manner. The replacement property guarantees that logical equivalence relative to any given set of equivalences (which can be viewed as equations) is a congruence relation on the formula algebra. Taking the quotients of the formula algebra by these congruences we obtain a class of algebras whose equational logic provides a complete algebraic representation of the original logic. This is the classical *Lindenbaum-Tarski process*, and it is used to generate the algebras traditionally associated with a number of different logics: the classical propositional calculus (Boolean algebras), the intuitionistic propositional calculus (Heyting algebras), modal logic (modal algebras), predicate logic (cylindric algebras), etc.

Traditional algebraic logic is mainly concerned with the algebraic theory of the various classes of algebras that arise in this way. *Abstract Algebraic Logic* on the other hand focuses on the process of algebraization itself and the questions that naturally arise from it. How does one abstract the Lindenbaum-Tarski process to logics without a biconditional or even a conditional? What are the limits of the algebraization process—more specifically, can the class of algebraizable logics be precisely characterized? As might be expected, several different answers to this question have been proposed, and this has led to a hierarchy of logics based on how tightly bound they are to the algebras that arise from them by the abstract Lindenbaum-Tarski process. The hierarchy has been intensively studied over the last several years and the results of these investigations will be surveyed in this talk.

▶ ANDRZEJ ROSŁANOWSKI, *How much sweetness is there in the Universe?*
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Since the very beginning of modern Set Theory there has been an interest in the *countable chain condition* (ccc) of Boolean algebras, topologies, forcing notions etc. The natural question of preserving ccc under various operations, most notably products, led to several *strong ccc* properties.

Amalgamation of forcing notions can be thought of as a generalization of a product, and this could be a good reason to ask for strong variants of ccc (for forcing notions) preserved under amalgamation. But the real interest in amalgamation comes from its use in building homogeneous forcing notions and consistency results concerning regularity of definable sets of reals.

In 1970, Robert M. Solovay proved that the existence of an inaccessible cardinal implies the consistency of "all projective sets of reals have Baire property and are Lebesgue measurable". Then Saharon Shelah showed that to get Lebesgue measurability one really needs inaccessibles, while the suitable forcing notion for the property of Baire can be constructed in ZFC. The proofs were built upon an analysis what happens if one tries to amalgamate forcing notions over random or Cohen forcing notion, respectively. The reason why we do need an inaccessible cardinal for Lebesgue measurability is that the amalgamation of the Amoeba for Measure forcing notion over a random real may fail ccc (and even may collapse $\aleph_1$). What allows us to amalgamate forcing notions without collapsing cardinals are strong ccc properties like *sweetness*.

We will discuss various friends and relatives of sweetness, how many sweet forcing notions are there and what we can use them for. We will also look at the opposite end of the spectrum: *sour forcing notions*.