Abstracts of invited talks in the Special Session on Computability Theory and Effective Mathematics

▸ PETER A. CHOLAK, MARIAGNESE GIUSTO, JEFFRY L. HIRST AND CARL G. JOCKUSCH, JR., *The logical strength and effective content of the thin set theorem and the free set theorem.*
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Let $[A]^k$ denote the set of all k-element subsets of the set A. The *thin set theorem*, due to Harvey Friedman, (see [2] or [1]), asserts that for every $k \in \omega$ and every function $f : [\omega]^k \to \omega$ there is an infinite set $A \subseteq \omega$ such that $f([A]^k) \neq \omega$. The *free set theorem*, also due to Harvey Friedman (see [2] or [1]), asserts that for every k and every function $f : [\omega]^k \to \omega$ there is an infinite set $A \subseteq \omega$ such that, for all $D \in [A]^k$, if $f(D)] \in A$, then $f(D) \in D$. We partially analyze the strength of these two results (and especially their restrictions to fixed k) in the sense of reverse mathematics. We also study the effective content of these results for fixed k, and compare them with corresponding results for Ramsey's Theorem [3].

[1] Peter A. Cholak, Mariagnese Giusto, Jeffry L. Hirst, and Carl G. Jockusch, Jr., *Free sets and reverse mathematics*, submitted for publication (available online at www.math.uiuc.edu/~jockusch/ under "Online Articles").

[2] Harvey Friedman and Stephen G. Simpson, *Issues and problems in reverse mathematics*, ***Computability theory and its applications (Boulder, CO, 1999)***, American Mathematical Society, Providence, RI, 2000, pp. 127–144.

[3] Carl G. Jockusch, Jr., *Ramsey's Theorem and recursion theory*, ***The Journal of Symbolic Logic***, vol. 37 (1972), pp. 268–280.

▸ PETER CHOLAK AND LEO HARRINGTON, *On the complexity of orbits in* $\mathcal{E}^*$.
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Using coding and other methods we will explore the orbits of certain definable sets. For example, we will show that a pair of hhsimple sets are automorphic iff they are Δ^0_3-automorphic, a pair of simple sets are automorphic iff they are Δ^0_6-automorphic, and, perhaps if there is time, a pair of promptly simple sets are automorphic iff they are Δ^0_3-automorphic. We will reflect on what these results tell us about our goal to construct an c.e. A where membership in A's orbit is Σ^1_1-complete.

▸ MALGORZATA DABKOWSKA, MIECZYSLAW DABKOWSKI AND VALENTINA S. HARIZANOV, *Degrees of the isomorphism types of countable structures.*
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We consider only countable models of theories in finite languages. The Turing degree of a structure $\mathcal{B}$ with domain ω, $\deg(\mathcal{B})$, is the Turing degree of its atomic diagram. The *Turing degree spectrum* of a structure $\mathcal{A}$ is $DgSp(\mathcal{A}) = \{\deg(\mathcal{B}) \colon \mathcal{B} \simeq \mathcal{A}\}$. A structure $\mathcal{A}$ is automorphically trivial if there is a finite subset S of its domain such that every permutation of the domain, whose restriction on S is the identity, is an automorphism of $\mathcal{A}$. Knight proved that for an automorphically nontrivial structure $\mathcal{A}$, $DgSp(\mathcal{A})$ is closed upwards. Hirschfeldt, Khoussainov, Shore and Slinko established that for every automorphically nontrivial structure $\mathcal{G}$, there is a symmetric irreflexive graph, a partial order, a lattice, a ring, an integral domain of arbitrary characteristic, a commutative semigroup, and a 2-step nilpotent group, $\mathcal{A}$, such that $DgSp(\mathcal{A}) = DgSp(\mathcal{G})$.

Jockusch defined the *Turing degree of the isomorphism type* of a structure $\mathcal{A}$, if it exists, to be the least Turing degree in $DgSp(\mathcal{A})$. Richter systematically studied degrees of the isomorphism types. She established that the Turing degree of the isomorphism type of a structure without a computable copy, which satisfies a certain condition on effective extendability of embeddings, does not exist. Such structures include linear orders and trees. A. N. Khisamiev has recently proved that they also include abelian p-groups. Richter introduced for a theory T a general combination method for constructing a model of T whose isomorphism types has arbitrary Turing degree. For example, there is such an abelian group. On the other hand, there is an abelian non-p-group whose isomorphism type does not have a degree. We extend Richter's method and apply it to investigate the isomorphism types of nonabelian *orderable groups*.

► NOAM GREENBERG, ANTONIO MONTALBAN AND RICHARD A. SHORE, *Generalized high degrees have the complementation property*.
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We show that if $\boldsymbol{d}$ is generalized high, then every degree below $\boldsymbol{d}$ has a complement in $D(\leq \boldsymbol{d})$. This answers a question raised by D. Posner in 1981.

► DENIS R. HIRSCHFELDT AND RICHARD A. SHORE, *Some combinatorial principles about linear orderings weaker than Ramsey's Theorem*.
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We analyze the status in terms of reverse mathematics of some combinatorial principles about linear orderings such as (ASDS): Every linear ordering has an infinite ascending sequence or an infinite descending one. All of the principles follow from Ramsey's theorem for pairs but none from WKL_0 (over RCA_0). In particular, we consider two, ASDS^* and $\mathrm{ASDS}^\#$ (analogous to SRT^2_2 and COH, respectively) which are incomparable over RCA_0 but which join up to ASDS in the sense that $\mathrm{RCA}_0 \vdash \mathrm{ASDS} \leftrightarrow (\mathrm{ASDS}^* \;\;\&\;\; \mathrm{ASDS}^\#)$ while $(\mathrm{RCA}_0 \;\;\&\;\; \mathrm{ASDS}^*) \nvdash \mathrm{ASDS}^\#$ and $(\mathrm{RCA}_0 \;\;\&\;\; \mathrm{ASDS}^\#) \nvdash \mathrm{ASDS}^*$. Most of our reverse mathematical conclusions follow from degree theoretic analyses.

► GEOFF LAFORTE, *Notions of randomness for reals*.
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Call a c.e. sequence of open sets $\langle U_n \colon n \in \omega \rangle$ a c.e. *test sequence* if for every n, $\mu(U_n) \leq 2^n$.

A real number is Martin-Löf random if it avoids (the intersection of) every c.e. test sequence. By Slaman and Kucera [3], a Martin-Löf c.e. random real must have complete Turing degree. Although Martin-Löf randomness is a natural notion, it has some drawbacks that led Schnorr [4] to introduce a less restrictive notion of randomness. Call a c.e. test sequence a Schnorr sequence if for all n, $\mu(U_n) = 2^n$. This stricter requirement on the test sequence leads to a weaker notion of randomness for reals. A c.e. real is Schnorr random if and only it has high Turing degree. Schnorr randomness also has interesting characterizations in terms of martingales and universal Turing machines. A similar, intermediate notion of randomness is computable randomness, which has a natural definition in terms of computable martingales, and alternate characterizations in terms of test sets and universal Turing machines.

Joint work with Rod Downey and Evan Griffiths.

[1] R. DOWNEY AND E. GRIFFITHS, *On Schnorr randomness*, extended abstract in ***Computability and Complexity in Analysis***, Malaga, (Electronic notes in Theoretical Computer Science, and proceedings, (Brattka, Schröder, Weihrauch, editors), Fern Universität 294-6/2002, 25-36), July 2002.

[2] R. DOWNEY, E. GRIFFITHS, G. LAFORTE, *On Schnorr and computable randomness*, in preparation.

[3] A. KUCERA, T. SLAMAN, *Randomness and recursive enumerability*, to appear in ***SIAM Journal of Computing***.

[4] C. SCHNORR, *Zufälligkeit und Warscheinlichkeit*, Lecture Notes in Mathematics, vol. 218, Springer-Verlag.

▸ TIMOTHY H. MCNICHOLL, *Automorphisms of the c.e. weak truth-table degrees*.
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We discuss work on constructing a non-trivial automorphism of the c.e. weak truth-table degrees.

▸ RUSSELL MILLER, *Spectra of structures and relations*.
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The *spectrum of a structure* $\mathcal{A}$ is the set of all Turing degrees of presentations of $\mathcal{A}$, i.e., $\{\deg(\mathcal{B}) \mid \mathcal{B} \simeq \mathcal{A}\}$. The *degree spectrum of a relation* R on a computable structure $\mathcal{C}$ is the set of Turing degrees of images of R under isomorphisms from $\mathcal{C}$ onto other computable structures. These notions are related conceptually, but we provide a concrete link by proving that for every linear order $\mathcal{L}$, there is a unary relation R on a fixed computable presentation $\mathbb{Q}$ of the rational numbers (as a linear order) such that the spectrum of $\mathcal{L}$ is precisely the degree spectrum of R on $\mathbb{Q}$.

The converse to this question remains open: given the degree spectrum of a unary relation R on $\mathbb{Q}$, is there a linear order $\mathcal{L}$ with that same spectrum? We offer some progress on this question by discussing when the degree spectrum of R on $\mathbb{Q}$ must be upwards-closed under Turing reducibility. (All spectra of infinite linear orders are upwards-closed, by a result of Knight.) We also note that in the related case of Boolean algebras, with the countable atomless Boolean algebra $\mathcal{B}$ in place of $\mathbb{Q}$, the converse fails: there exists a unary relation on $\mathcal{B}$ whose degree spectrum contains a low degree but does not contain 0, so that (by results of Downey and Jockusch) no Boolean algebra could have that spectrum.

We present joint work by Valentina Harizanov and the speaker.

▸ ANTONIO MONTALBÁN, *Embedding jump upper semilattices into the Turing degrees*.
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We prove that every countable jump upper semilattice can be embedded in $\mathcal{D}$. Where a jump upper semilattice (jusl) is an upper semilattice endowed with a strictly increasing and monotone unary operator that we call jump, and $\mathcal{D}$ is the jusl of Turing degrees. As a corollary we get that the existential theory of $\mathcal{D}(\leq, \vee, ')$ is decidable. We also prove that this result is not true about jusls with 0, by proving that not every quantifier free 1-type of jusl with 0 is realized in $\mathcal{D}$. On the other hand, we show that every quantifier free 1-type of jump partial ordering (jpo) with 0 is realized in $\mathcal{D}$. Moreover, we show that if every quantifier free type, $p(x_1, \ldots, x_n)$, of jpo with 0, which contains the formula $x_1 \leq 0^{(m)} and \ldots and x_n \leq 0^{(m)}$ for some m, is realized in $\mathcal{D}$, then every every quantifier free type of jpo with 0 is realized in $\mathcal{D}$.

► ANDRÉ NIES, *Randomness and lowness*.
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We relate randomness properties of reals and lowness properties, which say the real is computationally weak (a real is a subset of the natural numbers). First we consider r.e. randomness, also called Martin-Löf randomness. We show that a real A is far from random in the sense of having low initial segment complexity iff it is low for random, namely each random set is already random relative to A. This implies that the low for random reals are Turing below the halting problem, which had been asked by Terwijn, Kucera and others. Next we consider computable randomness, a notion weaker than ML-random which was introduced by Schnorr (1975). We answer a question of Ambos-Spies and Kucera by showing that the only low for computably random reals are the computable reals.

► REED SOLOMON, *On the filter of computably enumerable supersets of an r-maximal set*.
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One method for studying the algebraic structure of the computably enumerable sets under inclusion is to examine the principal filters. In this talk, we look at the principal filter generated by an atomless r-maximal set A. In particular, we consider the complexity of the index set of computable enumerable sets whose union with A is cofinite. We show that there is an atomless r-maximal set for which this index set is Σ^0_3 complete. The material presented in this talk is joint work with Steffen Lempp and Andre Nies.