

Abstracts of invited talks in the Special Session on Model Theory

- ▶ ITAY BEN-YAACOV, *Model theory of probability spaces*.
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Probability spaces, classically defined as sets equipped with a σ -algebra of subsets and a σ -additive positive measure, do not seem to admit a direct model theoretic treatment. Instead, we point out that for (almost) all practical purposes, probability spaces are equivalent to boolean algebras equipped with an additive positive measure, and show that a good model theoretic treatment exists for the category of these objects.

The framework in which this is done is that of cats (compact abstract theories), which are the analogues of elementary classes in a compact logic without negation. Many notions and tools, such as categoricity, stability and simplicity, generalise naturally from first order model theory have to this non-first-order context.

We prove that the category of complete atomless measured boolean algebras is a cat, and is ω -categorical and ω -stable as such. Non-dividing coincides of course with probabilistic independence. This allows us to translate probabilistic problems to model-theoretic terms, and attack them using ordinary stability theory.

- ▶ ALEXANDER J. BERENSTEIN, *Non-dividing in the Banach lattices $L^p(X, V, \mu)$* .
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We characterize non-dividing inside a saturated normed space structure (see [1]) $(L^p(X, V, \mu), +, \wedge, \vee, \|\cdot\|_p)$, where X is a measure space, V is a σ -algebra on X , μ is an atomless measure defined on V , $\|\cdot\|_p$ is the L^p norm and \wedge, \vee are the lattice operations (pointwise minimum and maximum). We include a second sort for the real field and the natural action from the real numbers on the space $L^p(X, V, \mu)$. We analyze this (non first-order) structure using positive bounded formulas (see [1]) and types formed by these formulas. We also discuss canonical bases in this setting.

This is a joint work with Itay Ben-Yaacov and C. Ward Henson.

[1] C. WARD HENSON AND JOSÉ IOVINO, *Ultraproducts in analysis*, *Analysis and Logic*, London Mathematical Society Lecture Notes Series 262, Cambridge University Press, 2002, pp. 1–113.

- ▶ ALFRED DOLICH, *On properties guaranteeing the existence of universal models in an elementary class*.
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In [1] Džamonja and Shelah give a criterion sufficient to guarantee that an abstract elementary class has universal elements under appropriate set theoretic conditions. Shelah in [2] and [3] has shown that the elementary class of any simple theory as well as several non-simple examples meet this criterion. In this talk we present a general framework which when present is sufficient to guarantee that an elementary class meets the criterion for the existence of universal models. This framework relies on the existence of independence notions for the theory satisfying axioms like those, though considerably weaker, satisfied by dividing in a simple theory. We present several examples of theories fitting into this framework.

[1] M. DŽAMONJA AND S. SHELAH, *On the existence of universal models*, Preprint.

[2] S. SHELAH, *Toward classifying unstable theories*, *Annals of Pure and Applied Logic*,

vol. 80 (1996), pp. 229–255.

[3] S. SHELAH, *The universality spectrum: consistency for more classes*, *Combinatorics, Paul Erdős is Eighty*, vol. 1 (1993), pp. 403–420.

- ▶ CHRIS MILLER, *Proper analytic subrings of the reals have Hausdorff dimension zero*.
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Every proper subring of the ring of real numbers that is analytic (in the sense of descriptive set theory) has Hausdorff dimension zero. Extensions of the method of proof yield that every proper analytic subring of the complex numbers either has Hausdorff dimension zero or is equal to the real numbers, and every proper analytic subring of p -adic numbers either has Hausdorff dimension zero or is equal to the p -adic integers.
This is a joint work with G. A. Edgar.

- ▶ ZIV SHAMI, *On Kueker simple theories and small unidimensional simple theories*.
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A countable theory T is said to be Kueker if it is not \aleph_0 -categorical and every uncountable model of T is \aleph_0 -saturated. Kueker's conjecture states that every Kueker theory is \aleph_1 -categorical. This conjecture has been proved by Buechler for superstable theories and by Hrushovski for stable theories. Here we show a partial 'simple' analog of the gap between Buechler's and Hrushovski's results. More precisely, we show that a Kueker simple theory eliminates \exists^∞ and densely interpret weakly minimal formulas (i.e., every non-algebraic formula is implied by a weakly minimal one). As part of the proof, we generalize Hrushovski's dichotomy for almost complete formulas to simple theories. We also conclude that a small unidimensional simple theory is supersimple of finite SU -rank.

- ▶ PATRICK SPEISSEGER, *A geometric proof of the definability of Hausdorff limits*.
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We give a geometric proof of the following well-established theorem for o-minimal expansions of the real field: the Hausdorff limits of a compact, definable family of sets are definable. While previous proofs of this fact relied on the model-theoretic compactness theorem, our proof explicitly describes the family of all Hausdorff limits in terms of the original family.
Joint work with Jean-Marie Lion.

- ▶ MONICA VANDIEREN, *Stability in Tame AECs*.
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We introduce the most general framework (to date) of Abstract Elementary Classes (AECs) for which a decent stability theory has been developed, namely tame AECs. Tame classes include (but are not limited to) homogeneous classes, excellent classes, categorical AECs with amalgamation and good frames. In this talk we summarize known stability results for tame classes and discuss open problems in this virgin territory.
This is joint work with Rami Grossberg.

- ▶ EVGUENI VASSILIEV, *Lovely pairs of simple structures*.

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We will survey some recent results on elementary pairs of models of simple theories. In a joint work with Itay Ben-Yaacov and Anand Pillay [1] we introduce the notion of a *lovely pair* of models of a simple theory, which generalizes the “beautiful pairs” of stable structures introduced by Poizat in [3], and the “generic pairs” of SU-rank 1 structures ([4]). A pair (M, P) of models of a simple theory T (where $P(M) \preceq M$) is said to be *lovely*, if for any type $p \in S_T(A)$ over a small subset A of M , p has a non-forking extension over $P(M) \cup A$ realized in M (extension property), and whenever p does not fork over $A \cap P(M)$, p is also realized in $P(M)$ (coheir property). For any simple theory T , lovely pairs of models of T exist, are elementarily equivalent, and in the stable case essentially coincide with the beautiful pairs. Assuming T satisfies certain “simple analogue” of the non-fcp property, T_P is again simple, forking in T_P has a natural description in terms of forking in T , and certain properties of T can be characterized in terms of T_P . For example, in the case when T is ω -categorical, T_P is ω -categorical if and only if T is one-based. In [2] we show that for a stable $T = T^{eq}$ without fcp, T_P has elimination of imaginaries if and only if there is no infinite group definable in T . We will also mention some applications of lovely (generic) pairs in the study of the geometry of SU-rank 1 structures.

[1] ITAY BEN-YAACOV, ANAND PILLAY, EVGUENI VASSILIEV, *Lovely pairs of models*, to appear in *Annals of Pure and Applied Logic*.

[2] ANAND PILLAY, EVGUENI VASSILIEV, *Imaginaries in beautiful pairs*, submitted, 2003.

[3] BRUNO POIZAT, *Paires de structures stables*, *The Journal of Symbolic Logic*, vol. 48 (1983), no. 2, pp.239–249.

[4] EVGUENI VASSILIEV, *Generic pairs of SU-rank 1 structures*, *Annals of Pure and Applied Logic*, vol. 120 (2003), no. 1–3, pp. 103–149.