Metamathematics I
Problem Set 7

Due: Friday November 30

Problems marked by $*$ are optional

1) a) (Reduction) Suppose A and B are recursively enumerable. Show that are are recursively enumerable sets $A_0 \subseteq A$ and $B_0 \subseteq B$ such that $A_0 \cap B_0 = \emptyset$ and $A_0 \cup B_0 = A \cup B$.

b) (Separation) Suppose A and B are Π_1 and $A \cap B = \emptyset$. Show that there is a recursive C such that $A \subseteq C$ and $B \cap C = \emptyset$. (HINT: Apply a) to $\neg A$ and $\neg B$.)

2. Show that $\{e : W_e \neq \emptyset\}$ is Σ_1-complete.

3. Let $Cof = \{e : \neg W_e \text{ is finite}\}$.
a) Show that Cof is Σ_3.
b)* Show that Cof is Σ_3 complete.

4. Suppose $X \subset \mathbf{N}^2$ is Δ_n. Show that there is a Δ_n set $Y \subseteq \mathbf{N}$ such that for all n, $Y \neq X_n$ (where $X_n = \{m : (n, m) \in X\}$). Thus there is no universal Δ_n set. (Hint: Consider $\{n : (n, n) \notin X\}$).