Errata for *An Invitation to Mathematical Logic*

David Marker

May 27, 2025

This Errata contains the mistakes I am aware of that are likely to confuse the reader. I do not list small typos that are less like to cause confusion. Hopefully both kinds of errors will be corrected in later printings of the book and in the on-line version.

Chapter 1

- pg 4 line-2: "binary relation" should be "binary operation"
- pg 9 line -5: The displayed formula should be

  $$\forall S \subset \mathbb{R}[(\exists x\ x \in S \wedge \exists y \forall x \in S\ x < y) \rightarrow$$
  $$\exists y(\forall x \in S\ x \leq y \wedge \forall z((\forall x \in S\ x \leq z) \rightarrow z \leq y))]$$

  The formula as stated says that $\mathbb{R}$ is well ordered.
- pg 11 line -5: Should be

  $$\phi \vee \psi \approx \neg(\neg\phi^* \wedge \neg\psi^*)$$
- pg 13 line -1: $\phi^* = \psi^* \vee \theta^*$.
- pg 17 lines 2 and 3: should be

  $$\begin{aligned} \mathcal{M} \models_\sigma \phi \quad &\Leftrightarrow \quad \mathcal{M} \models_\sigma \psi \text{ and } \mathcal{M} \models_\sigma \theta \\ &\Leftrightarrow \quad \mathcal{M} \models_\tau \psi \text{ and } \mathcal{M} \models_\tau \theta \end{aligned}$$

- pg 25 line 1: the first sentence should be "For any $x \in \mathbb{C}$ there is $y \in \mathbb{C}$ such that $y^2 = x^3 + 1$.

  line -3: Should be $X = \{n \in \mathbb{N} : \exists y_1 \ldots \exists y_m\ p(n, \overline{y}) = 0\}$,
- pg 26: Exercise 1.56 (c) The assertion that in a formula the counter is only zero at the end of the formula is false. Consider an atomic formula $f(x) = 0$ or the sentence $\exists x\ f(x) = 0$.

Chapter 2

- pg 33: Definition 2.4: $M \subset N$ should be $\mathcal{M} \subseteq \mathcal{N}$.
- pg 40: Exercise 2.24 should have the additional axiom

$$\forall x(x = 0 \vee \exists y\ s(y) = x \text{ and}$$

- pg 40: Exercise 2.25: The last sentence should say $\mathcal{M}_i \subseteq \mathcal{M}$ instead of $\mathcal{M}_i \subset \mathcal{M}$.
- pg 41: line 2: "collection" should be "chain"

Chapter 3

- pg 48: In rule Q1 we should also assume that no occurrence of a variable in t is bound in $\phi(t)$.

Chapter 4

- pg 57 line14 R^A should be $R^{\mathcal{M}}$
- pg 58 line -12:"constant symb not in Ł" should be "constant symbol not occurring in Γ"
- pg 58,59 The proof of Lemma 4.10: The display on page 59 should appear between the lines " There is a finite $\Delta \subseteq \Gamma$ such that $\Delta, \theta \vdash \psi$."

  and "Thus $\Delta \vdash \psi$ and $\Gamma \vdash \psi$, as desired."
- pg 60: The paragraph beginning on line -17 should read:

  Suppose Γ^* is inconsistent. Since proofs are finite, there are $\theta_1, \ldots, \theta_m \in \Gamma^*$ such that $\{\theta_1, \ldots, \theta_m\}$ is inconsistent. For each i, there is $\Gamma_i \in X$, such that $\theta_i \in \Gamma_i$. Since X is a chain, there is k such $\Gamma_i \subseteq \Gamma_k$. for all i. Thus all $\theta_i \in \Gamma_k$ and Γ_k is inconsistent, a contradiction. Hence $\Gamma^* \in P$.
- pg 61 last line of Exercise 4.16: $a \in K^*$ should be "$a \in K$
- pg 62 line 2 of Exercise 4.19: $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$
- pg 62 line -4: should be "T_1 and T_2 are inseparable"
- pg 63 in e) Let $\widehat{\mathcal{M}_i}$ be $\mathcal{M}_i$ viewed as an $\mathcal{L}_0$-structure.
- pg 63 line 18: explicitly should be "implicitly"

Chapter 5

- pg 69 lines 15–17 should say Let $\mathcal{L}^* = \mathcal{L} \cup \{c_\alpha : \alpha \in I\}$. Let

$$\Gamma^* = \Gamma \cup \{c_\alpha \neq c_\beta : \alpha, \beta \in I, \alpha \neq \beta\}.$$

Chapter 6

- pg 90 footnote 2 should be moved so that it appears at the end of the sentence Then $\mathcal{M} \models \psi([g])$, so $\mathcal{M} \models \phi$. in the following paragraph
- pg 93 footnote 3 should be:

  To see this, suppose, let ξ be a primitive 8^{th}-root of 1. Let $\alpha = \xi + \xi^{-1}$. Show that $\xi^{-2} = -\xi^2$ and conclude that $\alpha^2 = 2$. Next show that $\alpha^p = \alpha$ if $p \equiv \pm 1 \bmod 8$ and $\alpha^p \equiv \pm 3 \bmod 8$. Conclude that 2 is a square in $\mathbb{F}_p$ if and only if $p \equiv \pm 1 \bmod 8$.

Chapter 7

- pg 103 line -4 $\phi(v)$ should be $\psi(v)$
- pg 117 Exercise 17.42: L is a linear order, not necessarily a well order.
- pg 118 Exercise 17.47 a) Let $D = \ker \Psi$

Chapter 9

- pg 143: In Example 9.2 Instruction (8) should be $\mathtt{J}(\mathtt{1},\mathtt{1},\mathtt{5})$
- pg 144: In the explanation for Example 9.6 $r_1 \to r_3$ should be $r_1 \leftarrow r_3$
- pg 148: Exercise 9.18: the μ operator is introduced later on page 157
- pg 148 line-6: display should be $\prod_{i=0}^{k} p_i^{n_k+1}$
- pg 148 line-1: should say "$\ldots x \neq 1 \wedge \forall y(1 < y < x \to \neg(y|x)$"
- pg 150 line 2 $f_i(\overline{x}, m)$ is the largest n such that $p_i^n | F(\overline{x}, m)$.
- pg 154 line 11: display should be

$$(\overline{x}, y, s+1, \overline{x}, y, f(\overline{x}, y), 0, 0, \ldots).$$

- pg 156 line 3: should be $j(x, y, s) = 6$

Chapter 10

- pg 164 line -2 We code P by

$$\gamma(P) = \tau(\alpha(I_1), \ldots, \alpha(I_m)).$$

- pg 165 line -11: $N \leq j > 0$ should be $N \geq j > 0$
- pg 166 line 16: $\mathtt{T}(\mathtt{2},\mathtt{1})$ should be $\mathtt{T}(\mathtt{1},\mathtt{2})$
- pg 166 line -5 P_m should be Q_n
- pg 168 line 8: $\psi(x, x)$ should be $\Psi^{(2))}(x, x)$

Chapter 11

- pg 176 line 4: X should be f
- pg 177 line 12: should be "if $\phi_i(n)$ or $\phi_j(n)$
- pg 179 line-11: should be $e \in \neg K \Leftrightarrow g(e) \in Tot$.
- pg 181 should read $B = \{x : \exists y\ (x, y) \in Y\}$

Chapter 12

- page 190: Definition 12.5 Should read $A' = \{e : \phi_e^A(e)\downarrow\}$.
- page 195: line 10 should be $\sigma \in 2^{<\mathbb{N}}$
- page 196 Definition 12.21 should say $\phi_e^\tau(n)\downarrow$, $\phi_e^\rho(n)\downarrow$
- page 196 Lemma 12.23 should say

  For any computable perfect tree t there is a computable perfect tree $t_1 \leq t$ such that either:

  i) no $t_1(\sigma)$ e-splits or

  ii) $t_1(\sigma, 0)$ and $t_1(\sigma, 1)$ e-split $t_1(\sigma)$ for all σ.
- page 197 line 5: should say where $t(\eta) = t_e(\sigma, \eta)$, then i) and ii) will hold.
- page 199 line 20: $f \in [T]$ should be $f \in [T_{e+1}]$.
- page 199 line 24 should be: Thus $\phi_e^f(e)\downarrow$.
- page 199: in the display $\phi_e(e)^f$ should be $\phi_e^f(e)$
- page 199 in the last line of footnote 6 $[T_n]$ should be $\bigcap[T_n]$.
- page 203 Exercise 12.35 b) All of the Ks should be As.

Chapter 13

- pg 216 line 9 should be

  $\exists u \exists v \big[\, (\exists w < u(\beta(u, v, y+1)_0 = w \wedge \phi(\overline{x}, w)) \wedge \forall i \leq y - 1$
- pg 217 line 9 $j : \mathbb{N} \to M$
- pg 222 line 4 should be $\ulcorner \exists v_i \phi \urcorner = \langle \ulcorner \exists \urcorner, \ulcorner v_i \urcorner, \ulcorner \phi \urcorner \rangle$.
- pg 225 Theorem 13.46 D2. delete "If"
- pg 226 D1 should be D2

Chapter 14

- pg 240 Exercise 14.7 display should be

$$\sum_{i=0}^{n} \binom{x}{i} M^i < M^{n+1}$$

- page 240 line -1: Should be

$$\exists M \exists u \left[M = 2^x \wedge u < M^y \wedge u + zM^y \equiv (1+M)^x \bmod M^{y+1} \right].$$

- pg 241 The display in lines 3–6 should be

$$x! \binom{a}{x} = a(a-1)\cdots(a-(x-1)) < a^x$$

and

$$a^x = a(a-1)\cdots(a-(x-1)) \frac{a}{(a-1)} \cdots \frac{a}{a-(x-1)} = x! \binom{a}{x} \frac{1}{(1-\frac{1}{a})\cdots(1-\frac{x-1}{a})}.$$

- page 241 line -1: should say $y = \mathrm{quo}(z, w)$
- pg 244 line -9 should be $p_z | (1 + (x+1)M$
- pg 249 line 11: $\alpha^- k$ should be α^{-k}

Chapter 15

- page 259 lines -8 and -7: Should be $\alpha = \omega^{\gamma_1} + \ldots + \omega^{\gamma_m}$ and $\delta = \omega^{\delta_1} + \ldots + \omega^{\delta_n}$
- page 260 line 5 should be $\alpha(n) = \beta + \omega^\delta \cdot n$,
- page 263 line -10 should be

$$402653184 \cdot 2^{402653184} = 3 \cdot 2^{402653211}$$

- page 278 line -10: $\sim \widehat{\psi}$ should be $\widehat{\psi}$

Chapter 16

- pg 294 line -10 $x \in I$ should be $y \in I$
- pg 294 lines -7–10 should be

  Let x_1 be the least element of X. Choose $s \geq x_1, m$. Choose $x_1 < x_2 < \ldots < x_s$ be in X and let $l > x_s$. Then $Y = \{x_1, \ldots x_s\}$ is homogeneous for f_l and $|Y| \geq m, \min Y$, a contradiction.

- pg 312 line -7 $p_n | a$ should be $p_n \not| a$

Appendix A

- pg 319 line -2 above the footnote should be $\{z \in x : \phi(z, y_1, \ldots, y_n)\}$
- pg 323 line -1: should say "But then $\gamma \in \gamma$ and ϵ is not a linear order, a contradiction."
- pg 325 line 9: at the end of Definition A.22 add : "We need the Axiom of Infinity to show 'that ω is a set."
- pg 326: Exercise A.30 (a): Consider the lexicographic ordering of $\alpha \times \beta$.
- pg 327 line -5 change Shröder to Schröder
- pg 328 line 1: should by $x \in A \cup \bigcup_{n=0}^{\infty} A_{2n} \setminus A_{2n+1}$
- pg 329 line 3 $B = \{a \in A : a \notin f(a)\}$.
- pg 333 line 17 "for all κ" should be "for all $\mu < \kappa$"
- pg 334 in Theorem A.63 κ must be singular