1) For each natural number $k$ consider the game $G(k)$ where we start with a pile of $k$ matches. Alice and Bob take turns removing matches from the pile with Alice going first. At each turn the player may remove either 1 or 2 matches from the pile. The person who removes the last match loses.

For example, in $G(3)$ Alice has a winning strategy. She removes 2 matches and then Bob is forced to remove the last match.

a) Draw the game tree for $G(6)$ and determine who has a winning strategy.

b) Draw the game tree for $G(7)$ and determine who has a winning strategy.

c)\(^1\) Come up with a conjecture about the values $k$ where Alice has a winning strategy. Prove this if you can.