**Axioms for Ordered Fields**

**Basic Properties of Equality**

- (reflexivity)$x = x$;
- (associativity) if $x = y$, then $y = x$;
- (distributivity) if $x = y$ and $y = z$, then $x = z$;
- (substitution) for any function $f$ if $x_1 = y_1, \ldots, x_n = y_n$, then $f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$, similarly, any predicate true of $x_1, \ldots, x_n$ is true of $y_1, \ldots, y_n$.

**Axioms about Addition and Multiplication**

i) (commutativity) $a + b = b + a$ and $ab = ba$;
ii) (associativity) $(a + b) + c = (a + b) + c$ and $a(bc) = (ab)c$;
iii) (distributivity) $a(b + c) = ab + ac$;
iv) (zero rule) $a + 0 = a$;
v) (unity) $a \cdot 1 = a$;
vi) (substraction) for any $a$ there is $x$ such that $a + x = 0$ (we call $x = -a$
vii) (division) if $a \neq 0$ there is $x$ such that $ax = 1$ (we call $x = a^{-1}$),

**Order Axioms**

viii) $0 < 1$
ix) (transitivity of order) if $a < b$ and $b < c$, then $a < c$;
x) (trichotemy) either $a = b$ or $a < b$ or $b < a$;
xi) (irreflexive) $a \not< a$;
xii) (addition law) if $a < b$ then $a + c < b + c$;
xiii) (multiplication law) if $a < b$ and $c > 0$, then $ac < bc$.

All of these axioms are true in the real numbers $\mathbb{R}$ or the rational numbers $\mathbb{Q}$.

All the axioms except vii) are true in the integers $\mathbb{Z}$.