

Math 215: Introduction to Advanced Mathematics
Midterm I–Study Guide

- The midterm exam will be on Friday October 12. The exam will cover chapters 1–7.
- The course web page contains a week-by-week syllabus
<http://www.math.uic.edu/~marker/math215-F12/wtow.html>
and a list of key concepts
<http://www.math.uic.edu/~marker/math215-F12/concepts.html>
that gives a more detailed description of the material you are responsible for.
- One good way to study is to work on the sample problems suggested on the course web page.
- If I ask you to prove something from the axioms for ordered fields, I will provide you with a copy of the axioms.

Sample Exam

- 1) Consider the statement: if $a > b$, then $f(a) > f(b)$.
 - a) What is the contrapositive of this statement?
 - b) What is the converse of this statement?
 - c) What is the negation of this statement.

- 2) a) Determine the truth table for
$$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$$
 - b) Find a simpler statement equivalent to $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$.

- 3) a) Decide if the following statements are true in the nonnegative integers $\mathbb{N} = \{0, 1, 2, \dots\}$. Justify your answers.
 - i) $\forall x \exists y \ x < y$;
 - ii) $\exists x \forall y \ x < y$;
 - iii) $\forall x \exists y \ x + x = y$;
 - iv) $\forall x \exists y \ y + y = x$;
 - v) $\forall x \exists y \forall z \ y$ is a power of 2 and if z is a power of 2 dividing x , then z divides y .
 - b) Write down the negations of statements i) and ii) and v).

4) Prove from the axioms for ordered fields that if $0 < x < y$, then $x^3 < y^3$.

5) Prove that if $a^2 \geq 7a$, then $a \leq 0$ or $a \geq 7$.

6) Prove that

$$\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

for all $n \geq 2$.

7) Prove that $A \subseteq B$ if and only if $A \cup B \subseteq B$.