Math 215: Introduction to Advanced Mathematics
Problem Set 10

Due: Friday November 22

Do the following problems from the text: pg 185. 19

1) a) Let $|X| = 3$ and $|Y| = 7$. How many functions $f : X \to Y$ are there? How many injections?

b) What is the coefficient of a^3b^{88} in $(a+b)^{91}$?

2) Suppose $X \subseteq Y$. Let

$$\mathrm{FPF}(X,Y) = \{f \in \mathcal{F}(X,Y) : \forall x \in X \ f(x) \neq x\}.$$

These are the *fixed point free functions.*

a) Suppose $|X| = m \geq 1$ and $|Y| = n \geq m$. Prove that

$$\mathrm{FPF}(X,Y)| = (n-1)^m.$$

[Hint: Prove this by induction on m.]

b) Suppose $|X| = m \geq 1$. What is the probability that a randomly chosen $f \in \mathcal{F}(X,X)$ is fixed point free. What happens to this probability as $m \to \infty$.

3) a) Suppose X and Y are disjoint sets. Let

$$\mathcal{A} = \bigcup_{i=0}^{k} \mathcal{P}_i(X) \times \mathcal{P}_{k-i}(Y) = (\mathcal{P}_0(X) \times \mathcal{P}_k(Y)) \cup (\mathcal{P}_1(X) \times \mathcal{P}_{k-1}(Y)) \cup \ldots \cup (\mathcal{P}_k(X) \times \mathcal{P}_0(Y)).$$

Let $F : \mathcal{A} \to \mathcal{P}_k(X \cup Y)$ be the function

$$F(A,B) = A \cup B.$$

Prove that F is a bijection.

b) Use a) to conclude that

$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i} \binom{n}{k-i}.$$

(5pt Bonus) Do Problem 20 on page 185.