

Math 215: Introduction to Advanced Mathematics

Midterm I–Study Guide

- The midterm exam will be on Friday October 4. The exam will cover chapters 1–6.
- The course web page contains a week-by-week syllabus
<http://www.math.uic.edu/~marker/math215-F13/wtow.html>
This is a good way of seeing which material you are responsible for.
- One good way to study is to work on the sample problems suggested on the course web page.

Key Concepts Chapters 1–6

- Chapter 1–The Language of Mathematics
 - mathematical statements, propositions, predicates
 - building compound statements using AND, OR, and NOT.
 - truth tables–using truth tables to prove that statements are equivalent
 - negating statements: not (P and Q) is equivalent to (not P or not Q);
not(P or Q) is equivalent to (not Q or not P)
- Chapter 2–Implication
 - implications, universal implications
 - negating an implication: not (P \Rightarrow Q) is equivalent to (P and not Q)
 - reading phrases like "P is necessary for Q", "P whenever Q"... as implications
 - converses
 - contrapositives
 - if and only if statements
- Chapter 3–Proofs
 - direct proofs
 - proof by cases
 - proof by proving contrapositive

- Chapter 4–Proof by Contradiction
 - contradictions
 - proof by contradiction
 - proving $P \Rightarrow Q$ by contradiction
 - proving $P \Rightarrow (Q \text{ or } R)$ by contradiction
 - the square root of 2 is irrational
 - there are infinitely many prime numbers
- Chapter 5–The Induction Principle
 - proof by induction
 - changing the base case
 - definition by induction
 - strong induction
- Chapter 6–The Language of Set Theory
 - sets
 - equality of sets: proving two sets are equal
 - subsets: proving one set is a subset of another
 - the empty set
 - union, intersection and difference of sets
 - Venn diagrams
 - power sets
 - complements

Sample Exam

- 1) Consider the statement: if $a > b$, then $f(a) > f(b)$.
- What is the contrapositive of this statement?
 - What is the converse of this statement?
 - What is the negation of this statement?

- 2) a) Determine the truth table for

$$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$$

- b) Find a simpler statement equivalent to $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$.

- 3) Sketch the proof the $\sqrt{2}$ is irrational.

- 4) Prove that an integer n is divisible by 10 if and only if n is divisible by 2 and n is divisible by 5.

- 5) Prove that

$$\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

for all $n \geq 2$.

- 6) Prove that $A \subseteq B$ if and only if $A \cup B \subseteq B$.