Math 215: Introduction to Advanced Mathematics
Midterm II–Study Guide

• The second midterm exam will be on Friday November 15. The exam will cover chapters 7–11 + chapter 12.1 + chapter 22.

• The course web page contains a week-by-week syllabus
`http://www.math.uic.edu/~marker/math215-F13/wtow.html`

• One good way to study is to work on the sample problems suggested on the course web page.

Key Concepts Chapters 7–12.1 + chapter 12

- Chapter 7–Quantifiers
  1. $\exists$ and $\forall$
  2. finding negations of statements with quantifiers
  3. proving universal and existential statements
  4. Cartesian products
  5. convergence of sequences
- Chapter 8–Functions
  1. functions, domains, codomains and images
  2. compositon
  3. graphs of functions and the formal definition of functions
- Chapter 9–Injections, surjections, bijections
  1. injection, surjections and bijections
  2. behavior under composition
  3. inverse functions, f has an inverse if and only if f is a bijection
  4. the image and preimage functions
- Chapter 10–Counting
  1. cardinalities of finite sets

2. the addition, multiplication and inclusion-exclusion principles

- Chapter 11–Finite Sets

  1. $|X| \leq |Y|$ if and only if there is an injection $f : X \to Y$
  2. the Pigeonhole Principle
  3. Dirichlet's Theorem
  4. If Y is finite and $f : X \to Y$ is an injection, then X is finite
  5. if X is finite and $f : X \to Y$ is a surjection, then Y is finite,

- Chapter 12–Counting Functions and Sets–12.1 only

  1. counting the number of functions $f : X \to Y$ when X and Y are finite
  2. countin the number of injections $f : X \to Y$ when X and Y are finite

- Chapter 22–Partitions and Equivalence Relations

  1. equivalence relations
  2. partitions

Sample Questions[1]

1) Define the following concepts:

a) The sequence $(a_n)_{n=1}^{\infty}$ converges to a.
b) $f : X \to Y$ is injective.
c) $\sim$ is an equivalence relation on X.
d) Π is a partition of X.

2) a) Decide if the following statements are true in the nonnegative integers $\mathbb{N} = \{0, 1, 2, \ldots\}$. Justify your answers.

i) $\forall x \exists y\ x < y$;
ii) $\exists x \forall y\ x < y$;
iii) $\forall x \exists y\ x + x = y$;
iv) $\forall x \exists y\ y + y = x$;
v) $\forall x \exists y \forall z$ (y is a power of 2 and if z is a power of 2 dividing x, then z divides y).

b) Write down the negations of statements i) and ii) and v).

3) Let $A = \{1, 2, 3, 4\}$, let $B = \{1, 2, 3, 4, 5\}$ and define $f : A \to B$ by

x	$f(x)$
1	2
2	2
3	3
4	5

a) What is $\overrightarrow{f}(\{1, 2\})$?
b) What is $\overleftarrow{f}(\{2, 4, 5\})$?

4) Decide if each of the following statement is TRUE or FALSE. If FALSE, give an example showing it is FALSE.

a) Suppose $f : X \to Y$ and $g : Y \to X$ such that $g \circ f = I_X$. Then g is the inverse of f.

b) For all sets A and B, if $A \subseteq B$, then $B^c \subseteq A^c$.

c) For all sets A and B, $|A \cup B| = |A| + |B|$

d) $\forall x \in \mathbb{Z} \exists y \in \mathbb{Z}\ y < x$.

[1] This sample is consdierably longer than the midterm will be.

e) If $f : X \to Y$ and $A, B \subseteq X$ and $A \subseteq B$, then $\overrightarrow{f}(A) \subseteq \overrightarrow{f}(B)$.

f) Suppose $\sim_1$ is an equivalence relation on X and $\sim_2$ is an equivalence relation on X. Let $x \sim y$ if and only if $(x \sim_1 y$ or $x \sim_2 y)$. Then $\sim$ is an equivalence relation.

5) Suppose $f : X \to Y$ is a bijection. Prove that $\overrightarrow{f} : \mathcal{P}(X) \to \mathcal{P}(Y)$ is a bijection.

6) Suppose $m, n > 0$. Let $F = \{f : \mathbb{N}_m \to \mathbb{N}_n$ such that $f(1) = 1\}$.

a) Prove there is a bijection between F and $\mathcal{F}(\mathbb{N}_{m-1}, \mathbb{N}_n)$.

b) What is $|F|$?

7) Using the definition of convergence, prove that the sequence $(\frac{1}{\ln(n+1)})_{n=1}^{\infty}$ converges to 0.