

Basic Proof Methods

Direct Proof: To prove an implication $P \Rightarrow Q$, assume P and derive Q .

$$\frac{\text{Assume} \mid \text{Goal}}{P \mid Q}$$

Proof by Cases: To prove $(P \text{ or } Q) \Rightarrow R$, prove $P \Rightarrow R$ and $Q \Rightarrow R$.

case 1:

case 2:

$$\frac{\text{Assume} \mid \text{Goal}}{P \mid R}$$

$$\frac{\text{Assume} \mid \text{Goal}}{Q \mid R}$$

Proving “and” statements: To prove $P \Rightarrow (Q \text{ and } R)$, prove $P \Rightarrow Q$ and $P \Rightarrow R$.

$$\frac{\text{Assume} \mid \text{Goal}}{P \mid Q}$$

$$\frac{\text{Assume} \mid \text{Goal}}{P \mid R}$$

Proving the Contrapositive: To prove $P \Rightarrow Q$, it is equivalent to prove the contrapositive $((\text{not } Q) \Rightarrow (\text{not } P))$.

$$\frac{\text{Assume} \mid \text{Goal}}{\text{not } Q \mid \text{not } P}$$

Proof by Contradiction: To prove P , assume not P and prove any contradiction (Q and $(\text{not } Q)$).

$$\frac{\text{Assume} \mid \text{Goal}}{\text{not } P \mid \text{contradiction}}$$

Proving “or” statements: To prove $P \Rightarrow (Q \text{ or } R)$, proceed by contradiction. Assume P , not Q and not R and derive a contradiction.

$$\frac{\text{Assume} \mid \text{Goal}}{P, \text{ not } Q, \text{ not } R \mid \text{contradiction}}$$

Proofs of “if and only if”s: To prove $P \Leftrightarrow Q$. Prove both $P \Rightarrow Q$ and $Q \Rightarrow P$.

$$\frac{\text{Assume} \mid \text{Goal}}{P \mid Q}$$

$$\frac{\text{Assume} \mid \text{Goal}}{Q \mid P}$$