

Math 330: Abstract Algebra
SAMPLE MIDTERM

- 1) Define the following concepts:
 - a) subgroup;
 - b) even permutation;
 - c) homomorphism;
 - d) left coset.

- 2) State the following theorems:
 - a) Fundamental Theorem for Cyclic Groups;
 - b) Lagrange's Theorem;
 - c) First Isomorphism Theorem for Groups.

For Problems 3)–6) you must justify your answers to receive full credit

- 3) a) Find all subgroups of \mathbb{Z}_{36} . How many subgroups are there?
b) How many homomorphisms $\phi : \mathbb{Z}_{36} \rightarrow \mathbb{Z}_{42}$ are there?

- 4) In S_8 let α be the permutation

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 6 & 7 & 3 & 5 & 1 & 2 \end{bmatrix}$$

- a) Find α^{-1} .
 - b) What is $|\alpha|$?
 - c) Is $\alpha \in A_8$?
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- 5) How many elements of $D_3 \oplus \mathbb{Z}_6$ have order 2, order 3,4,5,6?

 - 6) Suppose $\phi : G \rightarrow H$ is a homomorphism. Prove that the kernel of ϕ is a normal subgroup of G .